Abstract

Contents

1 James Glimm Lecture

online (zoom)
ID: 81616498506
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July 3, Wednesday, 09:00-09:50

James Glimm and Shock Wave Theory

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The 1965 paper of James Glimm on hyperbolic conservation laws inaugurates the modern shock wave theory. This classical paper inspires many works, starting with the difficult Glimm-Lax paper. There is now a well-posedness theory for system of hyperbolic conservation laws in 1-D. The unique features of this fascinating development should have broad appeal in nonlinear analysis and PDEs. There is also the obvious question of how to generalize the Glimm theory to multi-D. This lecture will explain some of the main features of the Glimm theory, reminisce about related happenings, and make suggestions on possible ways for generalization of the Glimm theory.

2 Peter Lax Award Lecture

Chan Sui Kao Hall 100
July 3, Wednesday 09:50-10:40

Structure preserving high order Lagrangian schemes for the solution of hyperbolic equations with applicability from fluid-dynamics to astrophysics

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In this talk, we present a novel family of high order accurate numerical schemes for the solution of hyperbolic partial differential equations (PDEs) which combines several geometrical and physical structure preserving properties.

Indeed, first, we settle in the Lagrangian framework, where each element of the mesh evolves following as close as possible the local fluid flow, so to reduce the dissipation at contact waves and moving interfaces and to respect Galilean and rotational invariance. In particular, we choose the direct Arbitrary-Lagrangian-Eulerian setting which, in order to always guarantee the high quality of the moving mesh, allows to combine the Lagrangian motion with mesh optimization techniques. Our polygonal tessellation is thus regenerated at each time step, the previous one is connected with the new one by space-time control volumes, including hole-like sliver elements in correspondence of topology changes, over which we integrate a space-time divergence form of the original PDEs through a high order accurate ADER discontinuous Galerkin (DG) scheme. Mass conservation and the respect of the GCL condition are guaranteed by construction thanks to the integration over closed control volumes, and robustness over shock discontinuities is ensured by the use of an *a posteriori* sub-cell finite volume (FV) limiter.

In addition, our schemes are able to guarantee the exact preservation, up to machine precision, of equilibria and involution constraints: this allows us to obtain stable and robust simulations even for very complex physical models. To prove the capabilities of our novel approaches, we will show a wide set of numerical results ranging from compressible fluid-dynamics over classical magnetohydrodynamics up to the study of the Einstein-Euler field equations of general relativity.

E. Gaburro gratefully **acknowledges** the support received from the European Union with the ERC Starting Grant ALcHyMiA (No. 101114995).

3 Plenary Lectures

Chan Sui Kao Hall 100

July 5, Friday, 09:00-09:50

Approximation of non linear hyperbolic problems by globally continuous representation on polygons via virtual finite element techniques.

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In a series of unpublished papers, P.L. Roe and his students have started to develop a new third order method able to work on triangular unstructured meshes, using a continuous representation of data and no Riemann solvers. The degrees of freedom are the vertices of the elements and the mid points of the edges, as well as the average value of the conserved variables. The integration in time is done by a variant of the method of characteristics that allow to evaluate the flux at several sub-time steps. This explains the name "Active Flux". This idea has then been developed by Roe and others, a very uncomplete list of references can be found at [1,2,3,4,5].

In this talk, I will make a personal review of this method using the material of [6,7], show how to develop higher than third order schemes on triangles following [8]. Then I will explain the connection with the approximation tools provided by the Virtual Finite elements spaces and show several numerical results that confirms the stability and the accuracy of the methods. Some perspective will be drawn.

This is a joint work with Yongle Liu (U. Zürich), Jianfang Lin (U. Zürich), W. Boscheri (U. Chambery, France) and W. Barsukow (U. Bordeaux, France).

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[8] Rémi Abgrall, Jianfang Lin, and Yongle Liu. Active Flux for triangular meshes for compressible flows problems, 2023., Arxiv 2312.11271

July 2, Tuesday, 09:00-09:50

Quantum Computation of partial differential equations and related problems

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Quantum computers have the potential to gain algebraic and even up to exponential speed up compared with its classical counterparts, and can lead to technology revolution in the 21st century. Since quantum computers are designed based on quantum mechanics principle, they are most suitable to solve the Schrodinger equation, and linear PDEs (and ODEs) evolved by unitary operators. The most efficient quantum PDE solver is quantum simulation based on solving the Schrodinger equation. It will be interesting to explore what other problems in scientific computing, such as ODEs, PDEs, and linear algebra that arise in both classical and quantum systems, can be handled by quantum simulation.

We will present a systematic way to develop quantum simulation algorithms for general differential equations. Our basic framework is dimension lifting, that transfers nonlinear PDEs to linear ones, and linear ones to Schrodinger type PDEs. For non-autonomous PDEs and ODEs, or Hamiltonian systems with time-dependent Hamiltonians, we also add an extra dimension to transform them into autonomous PDEs that have only time-independent coefficients, thus quantum simulations can be done without using the cumbersome Dyson's series and time-ordering operators. Our formulation allows both qubit and qumode (continuous-variable) formulations, and their hybridizations, and provides the foundation for analog quantum computing.

July 2, Tuesday 09:50-10:40

Random compressible fluid flows

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Mathematical models arising in science and engineering inherit several sources of uncertainties, such as model parameters, and initial or boundary conditions. To predict reliable results, deterministic models are insufficient, and more sophisticated methods are needed to analyze the influence of uncertainties on numerical solutions. In computational fluid dynamics, stochastic collocation or the Monte Carlo methods are typically used to quantify the propagation of data uncertainty. Despite the large popularity of these methods, their rigorous convergence analysis for compressible fluid flows was missing in general.

In this talk, we will review our recent results obtained for the random compressible Euler and Navier-Stokes systems. We suppose that the initial and boundary data as well as model parameters, such as the viscosity coefficients, are random variables. Consequently, a solution of the PDE system will be a random process. The stochastic collocation or the Monte Carlo methods are combined with a suitable deterministic discretization scheme, such as a finite volume method. Since the compressible Navier–Stokes and the Euler equations are not uniquely solvable in the class of global weak solutions, we cannot apply pathwise arguments to analyze the random equations. Instead, we apply intrinsic stochastic compactness arguments via the Skorokhod representation theorem and the Gyöngy–Krylov method. We study both the statistical convergence rates as well as the approximation errors. The convergence of the deterministic Navier-Stokes or Euler system is realized via dissipative solutions. Assuming that the numerical solutions satisfy in probability suitable conditions leading to a global regular solution, we prove that the Monte Carlo finite volume method as well as the stochastic collocation finite volume method converge to a statistical strong solution. The convergence rates of the finite volume and statistical methods are discussed as well. Numerical experiments will illustrate theoretical results.

This research was supported by the German Science Foundation (DFG) under the grants TRR146 "Multiscale Simulation Methods for Soft Matter Systems" and 525853336 - SPP 2410 "Hyperbolic Balance Laws: Complexity, Scales and Randomness" as well as by the Sino-German project number GZ1465.

July 1, Monday, 09:50-10:40

From Navier-Stokes to discontinuous solutions of the compressible Euler

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The compressible Euler equation can lead to the emergence of shock discontinuities in finite time, notably observed behind supersonic planes. A very natural way to justify these singularities involves studying solutions as inviscid limits of Navier-Stokes solutions with evanescent viscosities.The mathematical study of this problem is however very difficult because of the destabilization effect of the viscosities.

Bianchini and Bressan proved the inviscid limit to small BV solutions using the so-called artificial viscosities in 2004. However, until very recently, achieving this limit with physical viscosities remained an open question.

In this presentation, we will present the basic ideas of classical mathematical theories to compressible fluid mechanics and introduce the recent a-contraction with shifts method. This method is employed to describe the physical inviscid limit in the context of the barotropic Euler equation.

July 4, Thursday 09:00-09:50

Linear inviscid damping and enhanced dissipation for shear flows

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The inviscid damping and enhanced dissipation are two important stabilizing mechanisms that play a crucial role in determining the transition threshold of laminar flows at high Reynolds number regime. This lecture is to introduce several approaches to establish the inviscid damping and enhanced dissipation estimates for the linearized 2-D Navier-Stokes (Euler) equations around shear flows.

4 Invited Lectures

Chan Sui Kao Hall 100
July 3, Wednesday, 11:15-12:00

Hydrodynamic models for quantum fluids

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A quantum fluid is a system of interacting particles, exhibiting the effects of quantum statistics also at a macroscopic scale. Such models describe various physical phenomena, such as superfluidity, Bose-Einstein condensation, the interior of neutron stars (plasmas), or electron transport in semiconductors. The prototypical example, given by the so-called quantum hydrodynamics

(QHD) system, is described by a compressible, inviscid, barotropic flow, subject to a stress tensor depending on the particle density and its derivatives.
In this talk, I will first review some fundamental tools to analyze solutions of QHD systems and I will provide some existence results for global in time weak solutions. Then, I will also present some recent studies and related to stability properties and the asymptotic behavior of such solutions.
Based on some joint works with Pierangelo Marcati (GSSI) and Hao Zheng (CAS, AMSS).

East Middle Hall Building 1-106
July 3, Wednesday, 11:15-12:00

Structure Preserving schemes for overdetermined hyperbolic systems: asymptotic, algebraic and thermodynamics constraints

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In this talk, we consider the family of symmetric and hyperbolic thermodynamically compati- ble (SHTC) models introduced by Godunov in 1961, which are overdetermined by fulfilling total energy conservation as an additional equation that does not need to be explicitly solved. Further- more, this model is endowed with possibly stiff relaxation source terms as well as with geometric constraints and involutions. We introduce a class of numerical schemes that is proven to mimic some of the properties exhibited by the mathematical model at the continuous level. In particular, we propose new structure preserving methods for the Godunov-Peshkov-Romenski model, that accounts in one single unified mathematical formalism for a wide range of physical phenomena in continuum mechanics, spanning from ideal and viscous fluids to hyperelastic solids. We devise asymptotic preserving schemes in the stiff relaxation limits of the model, which permit to retrieve at the discrete level the Navier-Stokes-Fourier stress tensor, on moving and fixed unstructured meshes. We also design provably thermodynamically compatible methods, that preserves entropy for smooth solutions and satisfy an entropy inequality in the case of discontinuities. Finally, we provide a general framework to comply with algebraic constraints embedded in the governing equations, hence demonstrating that the Geometric Conservation Law can be respected for the first time on fixed polygonal meshes, allowing the mass equation to be eventually discarded and replaced by a pure geometric formulation. Analogies with the Lagrangian form of the equations on moving meshes will also be provided.

East Middle Hall Building 1-106
July 2, Tuesday, 11:15-12:00

High order conservative numerical methods for three-temperature radiation hydrodynamics

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The three-temperature (3-T) radiation hydrodynamics (RH) equations are widely used in modeling various optically thick high-energy-density-physics environments, such as those in astrophysics and inertial confinement fusion. In this talk, we will discuss the methodology to construct high order conservative numerical schemes solving the 3-T RH equations. Specifically, three new energy variables are defined first, in the form of which three energy equations of the 3-T RH equations are rewritten. The main advantage of this formulation is that it facilitates the design of a scheme with both conservative property and arbitrary high order accuracy. Based on the WENO reconstruction and the strong stability preserving (SSP) high order time discretizations, we design a class of high order conservative Lagrangian and Eulerian schemes both in space and time respectively. To determine the numerical flux for the conservative advection terms in the 3-T RH equations, we propose a HLLC numerical flux which is derived from the divergence theorem rigorously and satisfies positivity property for density and internal energies. Various numerical tests are given to verify the desired properties of our schemes such as high order accuracy, non-oscillation, conservation, adaptation to multi-material problems for the Lagrangian scheme and adaptation to large deformation fluid flow for the Eulerian scheme. This is a joint work with Nuo Lei and Chi-Wang Shu.

East Middle Hall Building 1-106
July 4, Tuesday, 09:55-10:40

Critical thresholds in pressureless Euler–Poisson equations with background states

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In this talk, we discuss the critical threshold phenomena in a large class of one-dimensional pressureless Euler–Poisson (EP) equations, with non-vanishing background states. First, we establish local-in-time well-posedness in proper regularity spaces, which are adapted for a certain *neutrality condition* to hold. Next, we study the critical threshold phenomena in the neutrality-condition-satisfying pressureless EP systems, where we distinguish between two cases; attractive and repulsive forces. As an application, we analyze the critical thresholds for the damped EP system for cold plasma ion dynamics, where the density of electrons is given by the *Maxwell–Boltzmann relation*. This talk is based on a joint work with Dong-ha Kim, Dowan Koo, and Eitan Tadmor.

Chan Sui Kao Hall 100
July 1, Monday, 11:15-12:00

On Steady Transonic Shocks in a Finite Nozzle with Prescribed Pressure at the Exit

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In this talk, we are concerned with the inviscid 2-D steady Euler flow pattern with a single shock front in a finite nozzle, which enters the nozzle with a supersonic state and leave with a subsonic one. We are going to report our recent results on well-posedness of the transonic shock soluitons with the prescribed receiver pressure at the exit of the nozzle, as proposed by R. Courant and K.O. Friedrichs in their monograph “Supersonic Flow and Shock Waves”.

East Middle Hall Building 1-106
July 1, Monday, 11:15-12:00

Structure-Preserving Dynamical Low-Rank Methods for Kinetic Equations of Plasmas

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Dynamical low-rank methods have gained interest in recent years as a viable solution to the curse of dimensionality in the numerical solution of kinetic equations. Depending on the regime and nature of the solution, a direct application of these methods often results in high ranks or comes at the cost of losing physical properties such as conservation. Using the Vlasov-Fokker-Planck equation as a model problem, which is the governing equation for plasma dynamics, we demonstrate that when tuned to the problem's structure, the dynamical low-rank method can provide an efficient and robust approximation compared to the full tensor method. This talk is based on the joint work with Jack Coughlin and Uri Shumlak.

Chan Sui Kao Hall 100
July 2, Tuesday, 11:15-12:00

Analysis of hydrodynamic model of interacting agents

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Hydrodynamic models of interacting agents are systems of Partial Differential Equations describing the balances of mass and linear momentum for the continuum of interacting agents. The specific terms describing the interactions between individuals make these systems different from well-known fluid dynamics models, such as the compressible Euler or the Navier-Stokes equations. I will explain and explore these differences in the context of the existence theory of weak and measure-valued solutions.

In particular, I will introduce and discuss a generalization of the one-dimensional Aw-Rascle model of vehicular traffic, which has recently been proposed as a model for crowd dynamics. Mathematically, this system lies between the compressible Euler and compressible Navier-Stokes equations, featuring density-modulated dissipation. In one spatial dimension, the same system models the flow of rigid spheres surrounded by a viscous lubricant. At the level of classical solutions, the system is equivalent to the pressureless Navier-Stokes equations with the singular viscosity coefficient $\frac{\varepsilon}{1-\varrho}$. The first part of my talk will address the questions of existence, uniqueness, and the singular limit of weak and duality solutions as $\varepsilon \to 0$. I will then explain the differences and new challenges that arise in the analysis of this system in the multi-dimensional case. Here, we are able to prove the existence and weak-strong uniqueness of measure-valued solutions, as well as the ill-posedness of the model in the class of weak solutions. The latter result employs the convex integration technique, which has so far been restricted to inviscid compressible models. I will conclude the talk with a list of interesting open problems in this area.

This talk is based on several results obtained in collaboration with N. Chaudhuri (University of Warsaw), E. Feireisl (Czech Academy of Sciences), P. Gwiazda (Polish Academy of Sciences), M.A. Mehmood (Imperial College, London), L. Navoret (University of Strasbourg), and C. Perrin (CNRS, Aix Marseille Université).

Chan Sui Kao Hall 100
July 5, Friday, 09:55-10:40

Fluctuation theory at low density: a microscopic derivation

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I will review the state of the art in Grad's validity problem for a mathematical justification of fluid equations based on fundamental laws of classical mechanics. With the techniques currently available, such problem can be faced in some simple case for perfect gases, using the kinetic theory of Boltzmann as an intermediate step. In particular I will discuss a recent result establishing the connection between microscopic and hydrodynamic scales, for perturbations of a global equilibrium.

Chan Sui Kao Hall 100
July 4, Thursday, 09:55-10:40

Some recent studies on the subsonic boundary layers

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Even though there are extensive studies on the stability/instability of different hydrodynamic patterns in various physical settings, particularly in the high Reynolds number limit of laminar flows for the incompressible Navier-Stokes equations, there are much fewer mathematical results in the compressible setting. This talk will present a new approach to studying the compressible Navier-Stokes equations in the subsonic and high Reynolds number regimes. Some applications of this approach will also be discussed

5 July 1, Monday

Parallel Session A1 (CT)

Contributed Talks
East Middle Hall-Building 1-107,
14:00-18:05

Uniqueness and error estimates for hyperbolic conservation laws

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The talk will review some old and new results about uniqueness of solutions to hyperbolic conservation laws. In particular: for any n x n strictly hyperbolic system, any weak solution which takes values inside the domain of the semigroup of vanishing viscosity limits, and whose shocks satisfy the Liu admissibility conditions, actually coincides with a semigroup trajectory. Implications of his result toward a posteriori error estimates will be discussed.

Recent progress on compressible Euler equations

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I will discuss the recent progress on the singularity formation and global wellposedness on the compressible Euler Equations, and the inviscid limit from Navier-Stokes equations.

Relative Entropy Technique for Compressible Motions as Hamiltonian Flows with Respect to Placement and Momentum Maps

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In this presentation, we examine the application of the relative entropy technique to Hamiltonian flows that describe compressible motions in continuum mechanics. Traditional descriptions of classical mechanics for N particles employ Hamiltonians based on position and momentum vectors, whereas continuum fluid mechanics utilize Hamiltonian functionals based on density and momentum. For space dimensions $d \geq 2$, the Hamiltonian exhibits non-convex dependency on the deformation gradient or placement map, a characteristic influenced by material frame indifference, which poses inherent limitations for the application of the relative entropy technique.

Despite these limitations, our investigation clearly delineates the applications and boundaries of the technique, thereby maximizing its utility within the established theoretical framework. A key contribution of this work is the derivation of a relative Hamiltonian identity within the referential coordinate system, where the position and momentum fields are employed as primary and conjugate state variables, respectively.

This methodology, when applicable, yields strong stability estimates. For instance, We consider the Euler-Poisson systems in one space dimension. In the case of a particular pressureless model, our investigation verifies the non-increasing nature of L^2 state differences prior to the formation of δ-shocks. Additionally, we demonstrate weak-strong uniqueness, the stability of rarefaction waves, and the convergence to the gradient flow in the singular limit of large friction. Our study adapts its assumptions based on the presence or absence of pressure, thereby accommodating phenomena such as δ-shocks, vacuums, and shock discontinuities in weak solutions.

Particle paths for hyperbolic conservation laws

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We interpret nonlinear, scalar conservation laws

$$\partial_t u + \partial_x f(u) = 0, \qquad u_0 \in BV_{\text{loc}} \cap L^\infty(\mathbb{R})$$

as continuity equations, and couple the Kruzkhov uniqueness theory with the well-posedness of associated particle paths

$$\frac{d}{dt}x_t = \frac{f(u(x_t,t)) - f(c)}{u(x_t,t) - c}, \qquad x_0 \in \mathbb{R};$$

a system of ODEs which govern the mass transportation of the solution u. Thus, on the one hand we obtain a novel selection criterion for scalar conservation laws, and on the other hand we prove well-posedness and regularity with respect to the initial condition for a new class of ODEs. We also discuss applications. This is joint work with Ulrik Fjordholm and Ola Mæhlen (both at UiO).

Modeling Multi-lane Traffic Flow Models: from Micro to Macro

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This talk is devoted to the modeling and stability of vehicular multi-lane traffic flow in both microscopic and macroscopic frameworks. Firstly, we explore the dynamics of lane changing in microscopic variables, deriving simple lane changing conditions in order to extend the so-called 'Bando-Follow-the-Leader' one-lane model to the case of multi-lane flow [3]. The model derived is a microscopic hybrid model because the continuous dynamics of the equations are affected by the discrete events due to lane changes. Afterwards, we describe the derivation of novel first and second-order macroscopic multi-lane models that are obtained without assuming ad hoc micro-to-macro scale transitions. In particular, the first-order model [4] extends to the multi-lane case the famous LWR (Lighthill-Whitham-Richards) model [5,6], considering appropriate source terms which are strongly motivated by the microscopic dynamics and which are not modeled directly from the macroscopic point of view. The second-order model extends the ARZ (Aw-Rascle-Zhang) model [1,2], proposing a new system of balance laws including source terms that describe the evolution of the velocities, taking into account the contribution of lane changing. Furthermore, we investigate the equilibria for such models and establish conditions for their stability. Finally, we discuss numerical experiments conducted with realistic traffic scenarios

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Concentration and Cavitation in the Riemann solution to a macroscopic production model with van der Waals equation of state

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We are interested in studying the Riemann solutions and their limiting behavior for a macroscopic production model with van der Waals equation of state. First we construct the solution to the Riemann problem of the governing system which consists of only classical elementary waves, noting the presence of a vacuum state for specific initial data. Then in the limiting case we notice the formation of extreme concentration for a state variable in terms of Dirac delta distribution. Further it is observed that the delta shock solution of the governing system is different from that of pressureless gas dynamics system so a perturbation to the flux is made and the intrinsic phenomena of concentration and cavitation is examined in the limiting case. Additionally numerical simulations are performed to back up the analytical study.

Radon measure solutions of compressible Euler equations with applications to fluid-structure interaction problems

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In this talk I will review the progress on Radon measure-valued solutions to the compressible Euler equations for general fluids such as polytropic gases. Particularly, the results obtained by the speaker and his collaborators on inviscid hypersonic flows passing bodies and Newton-Busemann law, piston problems, and Riemann problems will be reported. These results demonstrate the necessity and wide applicability of measure solutions to hyperbolic conservation laws.

Global solutions of the one-dimensional compressible Euler equations with non-local interactions via the inviscid limit

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We are concerned with the global existence of finite-energy entropy solutions of the one-dimensional compressible Euler equations with (possibly) damping, alignment forces, and non-local interactions: Newtonian repulsion and quadratic confinement. Both the polytropic gas law and the general gas law are analyzed. This is achieved by constructing a sequence of solutions of the one-dimensional compressible Navier-Stokes-type equations with density-dependent viscosity under the stress-free boundary condition and then taking the vanishing viscosity limit. The main difficulties in this paper arise from the appearance of nonlocal terms. In particular, some uniform higher moment estimates for the compressible Navier-Stokes equations on expanding intervals with stress-free boundary conditions are obtained by careful design of the approximate initial data.

Parallel Session B1 (CT)

Contributed Talks
East Middle Hall-Building 1-109,
14:00-18:05

On the long-time behaviour of solutions to unforced evolution Navier-Stokes equations under Navier boundary conditions

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We study the asymptotic behaviour of the solutions to Navier-Stokes unforced equations under Navier boundary conditions in a wide class of merely Lipschitz domains of physical interest that we call *sectors*. The main motivations come from the celebrated results by Foias-Saut related to the long time behaviour of the solutions to Navier-Stokes equations under Dirichlet conditions.

Here the choice of the boundary conditions requires carefully considering the geometry of the domain Ω, due to the possible lack of the Poincaré inequality in presence of axial symmetries. In non-axially symmetric domains we show the validity of the Foias-Saut result about the limit at infinity of the Dirichlet quotient, in axially symmetric domains we provide two invariants of the flow which completely characterize the motion and we prove that the Foias-Saut result holds for initial data belonging to one of the invariants.

This is a joint work with Prof. Elvise Berchio (Politecnico di Torino, Italy) and Clara Patriarca (Université libre de Bruxelles, Belgium).

Global smooth solutions to compressible Navier-Stokes equations on bounded domains with large initial values

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This talk concerns the Vaigant-Kazhikhov model on general simply connected bounded domains with large initial values under the Navier-slip boundary conditions, where the viscosity coefficients are depended on the density. To get over the essential difficulties caused by the boundary, we first study the structure of Green's functions on the unit disc and establish the commutator estimates on it. Then we extend the arguments for general simply connected domains with the help of conformal mappings.

Uniqueness of weak solutions to the Maxwell–Stefan cross–diffusion system

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Cross–diffusion systems are strongly coupled parabolic systems describing phenomena in which multiple species diffuse and interact with one another, e.g. in fluid mechanics or population dynamics. Although many methods have been developed to study existence of weak solutions to such systems, uniqueness is in general an open problem. To this degree, we study a particular cross-diffusion system, known as the Maxwell–Stefan system which describes diffusive phenomena in a multicomponent system of gases. We employ renormalized solutions and give conditions under which such solutions are unique. We, then, study the relation between weak and renormalized solutions, allowing us to identify conditions that guarantee uniqueness of weak solutions. The proof is based on an identity for the evolution of the symmetrized relative entropy. Using the method of doubling the variables we derive the identity for two renormalized solutions and use information on the spectrum of the Maxwell–Stefan matrix to estimate the symmetrized relative entropy and show uniqueness.

Global Solutions of the Compressible Euler and Euler-Poisson Equations with Large Initial Data of Spherical Symmetry

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In this talk, we are concerned with the global existence theory for finite-energy solutions of the multidimensional compressible Euler equations and Euler-Poisson equations (both gaseous stars and plasmas are included) with large initial data of spherical symmetry. One of the main challenges is the strengthening of waves as they move radially inward towards the origin, especially under the self-consistent gravitational field for gaseous stars. A fundamental unsolved problem is whether the density of the global solution forms a delta measure (i.e., concentration) at the origin. We develop a new approach for the construction of approximate solutions as the solutions of an appropriately formulated problem for the compressible Navier-Stokes(-Poisson) equations with a carefully adapted class of degenerate density-dependent viscosity terms, so that a rigorous convergence proof of the approximate solutions to the corresponding global solution of the compressible Euler equations and Euler-Poisson equations with large initial data of spherical symmetry can be obtained. Even though the density may blow up near the origin at a certain time, it is proved that no delta measure (i.e., concentration) in space-time is formed in the vanishing viscosity limit for the finite-energy solutions of the compressible Euler-Poisson equations for both gaseous stars and plasmas in the physical regimes under consideration. The talk is based on joint works with G.Q. Chen, F.M. Huang, T.H. Li, L. He, W.Q. Wang, D.F. Yuan.

An inverse problem for supersonic potential flows past a cylindrically symmetric cone

Pu, Yun

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In this talk, we try to establish the existence of entropy solutions to the inverse problems for three-dimensional supersonic potential flows past a cone with a relatively large vertex angle. In this inverse problem, the uniform incoming flow and the pressure distribution on the surface of a cylindrically symmetric are prescribed, and we are required to find the generating curve of the cone and to determine the surrounding flow field. A modified Glimm scheme is applied to solve this inverse problem and the asymptotic behaviors of the entropy solution, the generating curve, and the strong shock are also studied.

Non-uniqueness for the hypo-viscous compressible Navier-Stokes equations

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We would discuss the Cauchy problem for the isentropic hypo-viscous compressible Navier-Stokes equations in all dimensions $d \geq 2$. For all hypo-viscosities $(-\Delta)^\alpha$ with $\alpha \in (0,1)$, we prove that there exist infinitely many weak solutions with the same initial data. This talk is based on a collaboration with Prof. Yachun Li, Prof. Deng Zhang and Dr. Zirong Zeng.

Spherically symmetric motions of the viscous heat-conducting and self-gravitating gas in around a rigid sphere

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We consider a system of equations describing spherically symmetric motions of the gas in around a rigid sphere. The gas is viscous, heat-conducting, self-gravitating and bounded by the free-surface. We first discuss the corresponding stationary problem and see the unique existence of a non-trivial steady flow. Subsequently, we will go into the time-global problem and get the asymptotic profile of the solution under a certain restricted but physically plausible condition on parameters and initial data.

Global regularity results of the 2D Boussinesq equations

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In this talk, I shall first introduce some global well-posedness results on the 2D Boussinesq equations, then I shall show our newest result on the 2D Boussinesq equations in the subcritical case. In addition, the global regularity of the critical case is also obtained provided that the L^∞-norm of initial temperature is small enough. The key ingredient and new observation are the iterative process between the combined quantity and the temperature, which may be of independent of interest. Finally, a global regularity result is also obtained for another subcritical case.

Instantaneous blowup of the entropy for the compressible Navier–Stokes equations

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This talk concerns the vacuum free boundary problem and the Cauchy problem (with far field vacuum) of non-isentropic compressible Navier-Stokes equations. It is proved that in several cases, with improper decaying rates of the initial density and temperature, the entropy of the classical solutions will blowup instantaneously.

These are joint works with Professor Zhouping XIN from the Chinese University of Hong Kong, Professor Jinkai LI from South China Normal University and Dr. Xin LIU from Texas A&M University.

Parallel Session C1 (CT)

Contributed Talks

East Middle Hall-Building 2-203,
14:00-18:05

Homogenized Equations for Fluid Dynamics with Periodic Modulation

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In the solution of first-order nonlinear hyperbolic PDEs, shocks form generically if the coefficients are constant, whereas solitary waves may arise if the coefficients are periodically-varying. The periodic variation in the coefficients introduces microstructure-heterogeneity into the system. These heterogeneities can induce effective dispersion that prevents the formation of shocks, instead leading to the formation of solitary waves. In this talk, our focus lies in analyzing the behavior of a fluid propagating in a one-dimensional narrow pipe with periodically-varying cross-sectional area, described by a system of first-order hyperbolic isothermal gas equations

$$(a\rho_t + (a\rho u)_x = 0,$$

$$(a\rho u)_t + (a\rho u^2 + ap(\rho))_x = p(\rho)a_x,$$

where ρ and u are the fluid density and velocity, and $p(\rho)$ is the pressure, given by

$$p(\rho) = \kappa\rho^\gamma.$$

The occurrence of these solitary waves is attributed to the coupling of nonlinearity and dispersive effects stemming from the medium's heterogeneity. We derive homogenized effective equations using multiple-scale perturbation theory which take the form of a constant-coefficient system including higher-order dispersive terms. We show that pseudospectral solutions to these equations agree well with direct solutions of the isothermal gas equation in periodically-varying cross-sectional area using a high-resolution finite-volume method.

On the effect of the Coriolis force on the enstrophy cascade.

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In this article, we investigate the effects of rotation on the dynamics, by neglecting stratification, in a 2D model where we incorporate the effects of the planetary rotation by adopting the β-plane approximation, which is a simple device used to represent the latitudinal variation in the vertical component of the Coriolis force.

We consider the well-known 2D β-plane Navier-Stokes equations (2DβNS) in the statistically forced case.

Our problem addresses energy-related phenomena associated with the solution of the equations. To maintain the fluid in a turbulent state, we introduce energy into the system through a stochastic force. In the 2D case, a scaling analysis argument indicates a direct cascade of enstrophy and an inverse cascade of energy. We compare the behaviour of the direct enstrophy cascade with the 2D model lacking the Coriolis force, observing that at small scales, the enstrophy flux from larger to smaller scales remains unaffected by the planetary rotation, confirming experimental and numerical observations. In fact, this is the first mathematically rigorous study of the above equations. In particular, we provide sufficient conditions to prove that at small scales, in the presence of the Coriolis force, the so-called third-order structure function's asymptotics follows the third-order universal law of 2D turbulence without the Coriolis force. We also prove well-posedness and certain regularity properties necessary to obtain the mentioned results.

The global Cauchy problem for the Euler-Riesz equations

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In this talk, we completely resolve the global Cauchy problem for the multi-dimensional Euler–Riesz equations, where the interaction forcing is given by $\nabla(-\Delta)^{-\sigma/2}\rho$ for some $\sigma \in (0, 2)$. We construct the global-in-time unique solution to the Euler–Riesz system in a H^s Sobolev space under a smallness assumption on the initial density and a dispersive spectral condition on the initial velocity. Moreover, we investigate the algebraic time decay of convergences for the constructed solutions. Our results cover the both attractive and repulsive cases as well as the whole regime $\sigma \in (0, 2)$. This is based on the collaboration with Y.-P. Choi and Y. Lee (Yonsei University).

Nonlinear stability of shock profiles to Burgers' equation with critical fast diffusion and singularity

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In this talk we propose the first framework to study Burgers' equation featuring critical fast diffusion in form of $u_t+f(u)_x = (\ln u)_{xx}$. The solution possesses a strong singularity when $u = 0$ hence bringing technical challenges. The main purpose of this talk is to investigate the asymptotic stability of viscous shocks, particularly those with shock profiles vanishing at the far field $x = +\infty$. To overcome the singularity, we introduce some weight functions and show the nonlinear stability of shock profiles through the weighted energy method. Numerical simulations are also carried out in different cases of fast diffusion with singularity, which illustrate and confirm our theoretical results. This talk is based on a joint work with Professors Jingyu Li, Ming Mei, and Jean-Christophe Nave.

Boundary layer problem and vanishing viscosity limit of the Hunter-Saxton equation

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The Hunter-Saxton equation models the propagation of weakly nonlinear orientation waves in a massive director field of a nematic liquid crystal. In this talk, I present some results about the boundary layer problem and vanishing viscosity limit of the Hunter-Saxton equation. These results are based on some joint works with Ming Mei, Jingyu Li and Kaijun Zhang.

Strong relaxation limits of some partially dissipative systems

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We develop a unified framework to investigate the relaxation limits of a class of partially dissipative systems in the whole space. Corresponding to the spectral analysis, we split the solution into low and high frequencies with a sharp parameter-dependent threshold in suitable hybrid Besov spaces, where the low frequencies are characterized by behaviors of the limit solution while the high frequencies equipped with critical norms decay fast as the relaxation parameter vanishes. This enables us to track the evolution of regularity properties of the global solution with respect to the relaxation parameter and establish explicit convergence rates of the singular limits. As applications, we investigate some relaxation limit processes, including chemotaxis system for vasculogenesis, two-phase fluid systems, Jin-Xin approximation and the Euler-Maxwell system.

Isothermal Limit of Entropy Solutions of the Euler Equations for Isentropic Gas Dynamics

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In this talk, we want to present the isothermal limit of entropy solutions in L^∞, containing the vacuum states, of the Euler equations for isentropic gas dynamics. First, We want to start with the explicit asymptotic analysis of the Riemann solutions containing the vacuum states. Then, we want to show the entropy solutions in L^∞ of the isentropic Euler equations converge strongly to the corresponding entropy solutions of the isothermal Euler equations, when the adiabatic exponent $\gamma \to 1$. This is achieved by combining careful entropy analysis and refined kinetic formulation with compensated compactness argument to obtain the required uniform estimates for the limit. The entropy analysis involves careful estimates for the relation between the corresponding entropy pairs for the isentropic and isothermal Euler equations when the adiabatic exponent $\gamma \to 1$. The kinetic formulation for the entropy solutions of the isentropic Euler equations with the uniformly bounded initial data is refined, so that the total variation of the dissipation measures in the formulation is locally uniformly bounded with respect to $\gamma > 1$. This is the joint work with Gui-Qiang G. Chen, and Fei-Min Huang.

Heat-conducting compressible fluids on time-dependent domains

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We consider a general compressible viscous, heat conducting fluid flow described by the Naiver-Stokes-Fourier system on domain which shape may change in time.

We concentrate on the existence of a variational solution to the system supplemented with slip boundary conditions for velocity and conservative boundary condition for the heat flux. The no-slip boundary condition for the velocity will be also discussed. The proof is based on a proper choice of the penalization. I will mentioned also the result in low Mach number regime.

This result is based on the joint works with O. Kreml, V. Machá, Š. Nečasová and is a part of the very recent book *Mathematical theory of compressible fluids on moving domains* (O. Kreml, V. Mácha, S. Nečasová, T. Piasecki, A. Wróblewska-Kamińska).

Incompressible limit of all-time solutions to isentropic Navier-Stokes equations with ill-prepared data in bounded domains

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In this paper, we study the incompressible limit of *all-time* strong solutions to the isentropic compressible Navier-Stokes equations with *ill-prepared* initial data and slip boundary condition in three-dimensional bounded domains. The uniform estimates with respect to both the Mach number $\epsilon \in (0, 1]$ and all time $t \in [0, +\infty)$ are derived by establishing a nonlinear integral inequality. In contrast to previous results for well-prepared initial data, the time derivatives of the velocity are unbounded which leads to the loss of strong convergence of the velocity. The novelties of this paper are to establish weighted energy estimates of new-type and to carefully combine the estimates for the fast variables and the slow variables, especially for the highest-order spatial derivatives of the fast variables. The convergence to the global strong solution of incompressible Navier-Stokes equations is shown by applying the Helmoltz decomposition and the strong convergence of the incompressible part of the velocity.

Parallel Session D1 (CT)

Contributed Talks
East Middle Hall-Building 2-303,
14:00-17:40

A multilayer shallow water model for tsunamis and coastal forest interaction

Bürger, Raimund

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Models and numerical methods of the impact of tsunamis on coastal forests are of vital importance for exploring the potential of coastal vegetation as a means of mitigation. Such a model is formulated as a multilayer shallow water system based on a free-surface formulation of the Euler equations for an ideal fluid. Specifically, the Euler equations are approximated by a layer averaged non-hydrostatic (LDNH) approach involving linear pressures, piecewise constant horizontal velocities and piecewise linear vertical velocities. Furthermore, based on [K. Iimura and N. Tanaka, Numerical simulation estimating effects of tree density distribution in coastal forest on tsunami mitigation, *Ocean Engrg.* **54** (2012) 223–232] drag forces, inertia forces, and porosity are added to model the interaction with the forest. These ingredients are specified in a layer-wise manner. Thus, the vertical features of the forest are described with higher accuracy than within a single-layer approach. Projection methods for the non-hydrostatic pressure in conjunction with polynomial viscosity matrix finite volume methods [M. J. Castro and E. Fernández-Nieto, A class of computationally fast first order finite volume solvers: PVM methods. *SIAM J. Sci. Comput.* **34** (2012) A2173–A2196] are employed for the numerical solution of the multilayer model, that is for the propagation of tsunamis and coastal flooding. Experimental observations and field data are used to validate the model. In general good agreement is obtained. Results indicate, moreover, that coastal vegetation can operate as an efficient natural barrier against coastal hazards and can significantly reduce the effects of tsunamis.

This presentation is based on joint work with Enrique D. Fernández-Nieto (Universidad de Sevilla, Spain) and Jorge Moya (Universidad de Concepción, Chile).

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An Energy Stable and Well-balanced Scheme for the Barotropic Euler System with Gravity under the Anelastic Scaling

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In this work we design and analyse an energy stable, well-balanced scheme for the Euler system with the gravitational potential, scaled by the Mach and the Froude numbers. The fully discrete scheme is semi-implicit in time and the finite volume space discretisation is employed using staggered marker and cell (MAC) grids. The conditional entropy stability of the scheme, under a CFL-like time step restriction, is achieved by introducing appropriate stabilisation terms in the convective fluxes. The stability estimate of the discrete relative entropy with respect to the hydrostatic steady state further ensures the well-balancing property of the scheme. The other relevant features of the scheme include the positivity of the density solution and the consistency of the approximate solutions with the weak solutions of the continuity equations. The asymptotic preserving (AP) property of the scheme, i.e. the boundedness of the discretisation parameters with respect to the diminishing Mach/Froude numbers, hence its consistency to the anelastic limit, is rigorously established. Several numerical case studies have been performed to showcase the efficiency and robustness of the proposed scheme.

A well-balanced second-order finite volume approximation for a coupled system of granular flow

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Abstract. In this work, a well-balanced second-order finite volume scheme is proposed and analyzed for a 2×2 system of non-linear partial differential equations. This particular system of equations describes the dynamics of growing sandpiles created by a vertical source on a flat, bounded rectangular table in multiple dimensions. To derive a second-order scheme, we combine a MUSCL type spatial reconstruction with strong stability preserving Runge-Kutta time stepping method, building upon the first-order scheme studied in (A. Aggarwal, A. Adimurthi, G. D. Veerappa Gowda, Godunov-type numerical methods for a model of granular flow, J. Comput. Phys., 305 (2016), pp. 1083–1118) and (Commun. Comput. Phys., 20 (2016), pp. 1071–1105). To ensure that the resulting scheme is well-balanced, we propose a modified limiting approach that allows the scheme to reduce to well-balanced first-order scheme near the steady state while maintaining the second-order accuracy away from it. The well-balanced property of the scheme is proven analytically in one dimension and demonstrated numerically in two dimensions. Additionally, numerical experiments reveal that the second-order scheme reduces finite time oscillations, takes fewer time iterations for achieving the steady state and gives sharper resolutions of the physical structure of the sandpile, as compared to the existing first-order schemes of the literature.
This is a joint work with A. Aggarwal and G. D. Veerappa Gowda

Well-balanced compact implicit numerical schemes for the shallow water equations with topography

Žeravý, Michal[1]; Frolkovič, Peter[2]; Parés, Carlos[3]

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In this work, we present a family of well-balanced high-resolution compact implicit finite difference numerical schemes for systems of balance laws that are unconditionally stable. The numerical schemes are derived using a Lax-Wendroff procedure in which the second-order time derivative is not fully replaced by the space ones, but the mixed space-time derivatives are used. Then, using an appropriate upwind approximation, the resulting algebraic system has a convenient form to be solved efficiently using the fractional time step and the fast sweeping method [1]. For the well-balanced property, following [2,3], the source term is rewritten as the difference of the flux function evaluated in local equilibrium. The methods are applied to the shallow water equations with topography: several numerical experiments that confirm the desired properties of the numerical schemes will be presented.

[1] Frolkovič, P., Žeravý, M.: *High resolution compact implicit numerical scheme for conservation laws.* Applied Mathematics and Computation, vol. 442, 2023.

[2] Parés, C., Parés-Pulido, C.: *Well-balanced high-order finite difference methods for systems of balance laws* Journal of Computational Physics, vol. 425, 2021.

[3] Castro Díaz, M.J., López-García, J.A., Parés, C.: *High order exactly well-balanced numerical methods for shallow water systems* Journal of Computational Physics, vol. 246, 2013.

An Asymptotic Preserving and energy stable scheme for the Euler-Poisson-Boltzmann system

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An asymptotic preserving (AP) and energy stable scheme for the Euler-Poisson-Boltzmann (EPB) system under the quasineutral scaling is designed and analysed. Appropriate stabilisation terms are introduced in the convective fluxes of mass and momenta, and the gradient of the electrostatic potential which lead to the dissipation of mechanical energy and consequently the entropy stability of solutions. The time discretisation is semi-implicit in nature, whereas the space discretisation uses a finite volume framework on a marker and cell (MAC) grid. The numerical resolution of the fully-discrete scheme involves two steps: the solution of a nonlinear elliptic problem for the potential and an explicit evaluation for the density and velocity. The proposed scheme possesses several physically relevant attributes, such as the positivity of density, entropy stability and the consistency with the weak formulation of the continuous EPB system. The AP property of the scheme, i.e. the boundedness of the mesh parameters with respect to Debye length and its consistency with the quasineutral limit system, is demonstrated. The results of numerical case studies are presented to substantiate the robustness and efficiency of the proposed method.

Active flux like method to shallow water models: well-balancing and arbitrarily high-order

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In this talk, I will introduce an arbitrarily high-order accurate fully well-balanced numerical method for the one-dimensional shallow water models, including the Saint-Venant systems of equations and blood flow model in an

artery. The developed method is based on a continuous representation of the solution and a natural combination of the conservative and primitive formulations of the studied PDEs. The degrees of freedom are defined as point values at cell interfaces and moments of the conservative variables inside the cell. The well-balanced property, in the sense that capable of exactly preserving both the zero and non-zero velocity equilibria, is achieved by a well-balanced approximation of the source term in the conservative formulation and a well-balanced residual computation in the primitive formulation. Several numerical tests are shown to prove its well-balanced and high-order accuracy properties.

Efficient Numerical Strategies for Hyperbolic Systems and Sediment Evolution in Shallow Water Models

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A comprehensive approach to enhance the stability and efficiency of numerical schemes for solving hyperbolic systems of balance laws with relaxed source terms and for computing sediment evolution in shallow water models has been proposed. Firstly, two semi-implicit-type second-order Compact Approximate Taylor (CAT2) schemes augmented with a local *a posteriori* Multi-dimensional Optimal Order Detection (MOOD) paradigm have been presented. These schemes exhibit high accuracy for smooth solutions, non-oscillatory behavior for irregular ones, and maintain positivity almost flawlessly. Numerical experiments on various test cases, including smooth and non-smooth initial conditions, demonstrate the effectiveness of the semi-implicit CATMOOD schemes, showcasing their accuracy and efficiency compared to a second-order semi-implicit Runge-Kutta (RK) method.

Secondly, the computational challenges in simulating sediment evolution within the Exner model for sediment transport in shallow water systems has been analyzed. Explicit methods suffer from severe stability constraints due to the fast velocity of free-surface waves, leading to prolonged computation times. To overcome this limitation, an Implicit-Explicit (IMEX) strategy to implicitly treat water waves while computing sediment evolution has been proposed. By doing so, the computational efficiency has been improved by enabling much larger time steps than those permitted by standard CFL conditions on explicit schemes.

By merging these two approaches, a unified framework for efficiently solving hyperbolic systems is offered. This integrated methodology promises improved stability, accuracy, and computational efficiency, making it a valuable tool for a wide range of applications in computational fluid dynamics and environmental modeling.

Semi-implicit fully exactly well-balanced schemes for the shallow water equations

Muñoz-Ruiz, María Luz

Caballero-Cárdenas, Celia

Castro-Díaz, Manuel J.

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In this work, semi-implicit finite volume schemes for the one-dimensional shallow water system are presented. These schemes are fully well-balanced, i.e., they preserve all the smooth steady states, not only the water-at-rest ones. Moreover, they are fully exactly well-balanced, that is, they exactly preserve the steady states and not just discrete approximations of them.

To obtain these schemes, relaxation and splitting techniques are combined in such a way that two systems are finally solved, one for pressure and the other for transport, so that acoustic and transport phenomena are decoupled.

The pressure system can be solved either explicitly or implicitly. The use of an implicit scheme is especially recommended in those situations where the Froude number is small, that is, when the fluid velocity is considerably smaller than the celerity. In such cases, the main constraint on the time step is given by the pressure term. Solving the pressure system implicitly allows us to consider larger time steps than those imposed by the CFL condition if the resolution is done explicitly. On the other hand, the resolution of the transport system will always be done explicitly.

The splitting technique presented avoids difficulties present in the use of Lagrangian-projection- type schemes previously used by the authors, and the proposed semi-implicit schemes preserve all the steady states without the need to solve the nonlinearities associated to the pressure. The only nonlinearities solved are those involved in the computation of the local steady states and the ones related to the transport step.

Here we propose first and second order schemes, for which we present several numerical experiments in order to test their accuracy and efficiency.

Parallel Session E1-1 (CT)

Contributed Talks
East Middle Hall-Building 1-201,
14:00-15:40

Optimal Control of the Generalized Riemann Problem for Hyperbolic Systems of Conservation Laws

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In this talk, we analyze optimal control problems for quasilinear strictly hyperbolic systems of conservation laws where the control is the initial state of the system. Similar problems for scalar conservation laws have already been studied. The case of hyperbolic systems is more involved due to the coupling of the characteristic fields. So far, it is known that the solution to a hyperbolic system of conservation laws is only directionally differentiable w.r.t. the initial state in a suitable sense, for example shown by Bressan and Marson (1995). However, these results are not strong enough to solve the optimal control problem discussed in this talk in a derivative-based way.

We begin our analysis by considering the Generalized Riemann Problem (GRP), which has a piecewise smooth initial state with exactly one discontinuity. For piecewise C^1 initial data we obtain the existence, uniqueness and stability of an entropy solution by a careful fixed point argument built on the associated Riemann Problem with piecewise constant initial states. The construction yields insights into the structure and regularity of the solution and provides a foundation to derive differentiability results of the control-to-state mapping.

The entropy solution is piecewise C^1. Its smooth parts are separated by C^2 curves which are either shock curves or boundaries of rarefaction waves. In a subsequent step, we show that these curves depend differentiably on the initial state. This allows the transformation to a reference space on a fixed space-time domain. In the reference space, we can show that the transformed solution depends differentiably on the initial state in the topology of continuous functions. This property can be translated back to the physical coordinates in the sense that the solution operator of the hyperbolic system into regions with a positive distance to shock curves or boundaries of rarefaction waves is continuously differentiable in the topology of continuous functions w.r.t. the initial state. For this, a detailed knowledge of the structure of the solution and the behaviour of the shock curves is crucial. As an immediate consequence, the differentiability of tracking type functionals for the optimal control problem follows.

Lyapunov Stabilization for Nonlocal Traffic Flow Models

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Using a nonlocal second-order traffic flow model, we present an approach to control the dynamics towards a steady state. The system is controlled by the leading vehicle, which travels at a prescribed speed and also determines the steady state. We consider both microscopic (trajectory based) and macroscopic (density based) scales. The fixed point of the microscopic traffic flow model is (locally) asymptotically stable for any kernel function. To obtain global stabilization, we present Lyapunov functions for both the microscopic and macroscopic scales and compute the explicit rates at which vehicles affected by nonlocality tend toward the stationary solution. We obtain stabilization results for a constant kernel function and arbitrary initial data or concave kernels and monotone initial data. In particular, the stabilization is exponential in time. Numerical examples will demonstrate the theoretical results.
This is a joint work with Michael Herty (RWTH Aachen) and Simone Göttlich (University of Mannheim).

A Variational Calculus for Optimal Control of Networks of Scalar Conservation or Balance Laws

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Networks of scalar conservation or balance laws provide models for vehicular traffic flow, supply chains or transmission of data. Such networks usually consist of initial boundary value problems (IBVPs) of scalar conservation or balance laws on every edge coupled by node conditions. For their optimal control a variational calculus is desirable that implies differentiability of objective functionals w.r.t. controls. In the last decade research on IBVPs successfully introduced a variational calculus which implies differentiability of objective functionals of tracking type and also yields an adjoint based gradient representation for the functional. This talk presents recent progress in an extension of these results to networks of scalar conservation or balance laws. Regarding node conditions we introduce a framework for their representation compatible with the known approach on single edges which allows us to extend the results such as continuous Fréchet differentiability for functionals of tracking-type and an adjoint based gradient representation on the network.

Stabilization of multi–dimensional hyperbolic systems

Thein, Ferdinand (joint work with Michael Herty)

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We are interested in the boundary feedback stabilization of multi–dimensional hyperbolic PDEs. Two cases are discussed in this talk. First we study systems with diagonal Jacobians which for example arise in the treatment of Hamilton-Jacobi equations, cf. [1].
The second case considers symmetric hyperbolic systems, cf. [2].
For each case a proper Lyapunov function is defined. With this we show stabilization in L^2 for the arising system using a suitable feedback control. We further present examples based on a forming process and on additive manufacturing for the first case and study the barotropic Euler equations for the second case.

[1] M. Herty and F. Thein. Stabilization of a multi-dimensional system of hyperbolic balance laws. *Mathematical Control and Related Fields*, 2023. `https://doi.org/10.3934/mcrf.2023033`

[2] M. Herty and F. Thein. Boundary feedback control for hyperbolic systems. *ESAIM: Control, Optimisation and Calculus of Variations (submitted)*, 2023. `http://arxiv.org/abs/2303.05598`

Parallel Session E1-2 (CT)

Sino-German session on advanced numerical methods for hyperbolic balance laws
East Middle Hall-Building 1-201,
16:00-18:05

Well-balanced path-conservative discontinuous Galerkin methods with equilibrium preserving space for two-layer shallow water equations

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This paper introduces well-balanced path-conservative discontinuous Galerkin (DG) methods for two-layer shallow water equations, ensuring exactness for both still water and moving water equilibrium steady states. The approach involves approximating the equilibrium variables within the DG piecewise polynomial space, while expressing the DG scheme in the form of path-conservative schemes. To robustly handle the nonconservative products governing momentum exchange between the layers, we incorporate the theory of Dal Maso, LeFloch, and Murat (DLM) within the DG method. Additionally, linear segment paths connecting the equilibrium functions are chosen to guarantee the well-balanced property of the resulting scheme. The simple "lake-at-rest" steady state is naturally satisfied without any modification, while a specialized treatment of the numerical flux is crucial for preserving the moving water steady state. Extensive numerical examples in one and two dimensions validate the exact equilibrium preservation of the steady state solutions and demonstrate its high-order accuracy. The performance of the method and high-resolution results further underscore its potential as a robust approach for nonconservative hyperbolic balance laws.

Investiagtion of isothermal Baer-Nunziato-type models

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Compressible multi-component flows where the fluid is a mixture of several components all of which may be present in different aggregate states have a wide range of applications, for instance a mixture of reacting gases or a mixture of a liquid and a gas. They have been successfully modeled by Baer-Nunziato-type models, see for instance [3]. In recent years, work on barotrop Baer-Nunziatio models has been published in [1,2,4].

In the current work the main interest is on multi-component Baer-Nunziato-type models for mixtures with an arbitrary number of components all of which are modeled by immiscible isothermal fluids. These are given by balance equations for volume fractions, density and momentum for each component accounting for the relaxation to equilibrium by means of relaxation terms.

First of all, mathematical properties of the models are discussed such as hyperbolicity, investigate the characteristic fields and derive the corresponding Riemann invariants. All of these properties are helpful for the numerical discretization of the models and to determine the solution of Riemann problems.

Then thermodynamical consisteny of the models is verified. For this purpose, first an appropriate Lax' entropy-entropy flux pair is derived where, in particular, the phasic energy equations including the heat flux is accounted for, see [3]. Furthermore, to ensure an entropy equality appropriate interfacial pressures and interfacial velocity are chosen. In particular, unique interfacial pressures are determined depending on the interfacial velocity chosen as a convex combination of the phasic velocities. By this, the Baer-Nunziato models are closed.

Finally, instantaneous relaxation to equilibrium is investigated and appropriate algorithms are presented. These are used to perform numerical computations using a path-conservative DG scheme.

This is joint work with
Maren Hantke, Martin-Luther-University Halle-Wittenberg, Germany,
Aleksey Sikstel, University of Cologne, Germany
Ferdinand Thein, RWTH Aachen University, Germany

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Convergence of a finite volume method for the Navier-Stokes-Fourier system with Dirichlet boundary conditions

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For compressible Navier–Stokes–Fourier (NSF) system the weak solutions are not known to be uniquely determined by given data. In the case of non-homogeneous Dirichlet boundary data, the situation is even worse due to the absence of conventional energy estimates.

We work with a newly developed concept named *ballistic energy*, and study the convergence of a finite volume method for the Dirichlet NSF problem. Specifically, we firstly propose a finite volume scheme, for which the unconditional positivity-preserving properties and ballistic energy balance are shown. Next, assuming that numerical solutions are uniformly bounded, we derive a priori estimates via ballistic energy balance, and then prove the consistency of our finite volume approximations. Finally, we obtain the weakly convergence of the numerical solutions towards a generalized, the so-called dissipative measure-valued, solution. Indeed, by applying the conditional regularity and weak-strong uniqueness results, we enhance the weak convergence to strong convergence (towards the strong solution). Numerical results for the Rayleigh-Bénard convection problem confirm our theoretical analysis.

A-posteriori error estimates for systems of hyperbolic conservation laws via computing negative norms of local residuals

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We provide rigorous a-posteriori error estimates for first order finite-volume approximations of nonlinear systems of hyperbolic conservation laws in one spatial dimension. Our estimators rely on recent stability results by Bressan, Chiri and Shen and a novel method to compute negative order norms of local residuals. Computing negative order norms becomes possible by suitably projecting test functions onto a finite dimensional space. Numerical experiments show that the error estimator converges with the rate predicted by a-priori error estimates. In addition, we discuss possible extensions to higher-order schemes, such as MUSCL.

A linear monolithic finite element method for fluid-shell interaction: stability and error estimates

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We propose a linear monolithic scheme for the approximation of a fluid-structure interaction problem, that is a deformable plate interacting with a viscous incompressible fluid. We take the backward Euler method for the time discretization, and use P1-bubble/P1/P1 elements for the approximation of the fluid velocity, pressure, and the structure displacement. We use the arbitrary-Lagrange-Euler method and work on the reference domain, meaning that re-meshing is not needed during the time evolution. We prove the stability of the numerical solution as well as the linear convergence rate with respect to the size of the time step Δt and mesh h. Finally, we present the numerical experiments to illustrate the theoretical results.

Parallel Session F1 (CT)

Contributed Talks
East Middle Hall-Building 1-202,
14:00-17:40

Uniform bound of the highest-order energy of the 2D incompressible elastodynamics

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This talk concerns the time growth of the highest-order energy of the systems of two dimensional incompressible isotropic Hookean elastodynamics. The two dimensional systems are nonlocal quasilinear wave equations where the unknowns has slow temporal decay. By observing an inherent strong null structure, the global well-posedness of smooth solutions near equilibrium was first proved by Zhen Lei where the highest-order generalized energy may have certain growth in time. We develop novel energy estimate strategies and show that the highest-order generalized energy is uniformly bounded for all the time.

Improved blowup time estimate for fourth-order damped wave equations with strain term at arbitrary positive initial energy

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By proposing a new differential inequality, we improve the upper bound of the blow-up time estimate for the nonlinear fourth-order damped wave equation with strain term at arbitrary positive initial energy. We also give two new initial conditions to expand the range of the initial data leading to the finite time blow-up of solutions. We obtain a sharp result of finite time blow-up for the special case of the new differential inequality. Some simulations exhibit and verify the main results.

On the critical regularity of nonlinearities for semilinear classical wave equations

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We consider the Cauchy problem for semilinear classical wave equations

$$u_{tt} - \Delta u = |u|^{p_{\mathrm{Str}}(n)}\mu(|u|)$$

with the Strauss exponent $p_{\mathrm{Str}}(n)$ and a modulus of continuity $\mu = \mu(\tau)$, which provides an additional regularity of nonlinearities in $u = 0$ comparing with the power nonlinearity $|u|^{p_{\mathrm{Str}}(n)}$. We obtain a sharp condition on μ as a threshold between global (in time) existence of small data radial solutions by deriving polynomial-logarithmic type weighted $L_t^\infty L_r^\infty$ estimates, and blow-up of solutions in finite time even for small data by applying iteration methods with slicing procedure. These results imply a conjecture for the critical regularity of source nonlinearities for semilinear classical wave equations. We verify this conjecture in the 3d case. This is a joint work with Prof. Michael Reissig (TU Bergakademie Freiberg).

Fick's law diffusion selects Neumann boundary condition, while Chapman's law diffusion selects Dirichlet boundary condition.

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When the diffusivity of particles is spatially heterogeneous, there have been many efforts to describe random diffusion by parabolic partial differential equations. There are three classical diffusion laws: Fick's Law, Wereide's Law, and Chapman's Law. Recently, a general diffusion law was derived that encompasses these three diffusion laws. In this study, we consider the spatial domain divided into two sub-domains: the inner domain and the outer domain. If we take the zero diffusivity limit in the outer domain, it means that there are no more dynamics in the outer domain, so the solution in the inner domain satisfies certain boundary conditions at the interface. However, different diffusion laws give different boundary conditions, and there is a critical diffusion law that distinguishes Neumann boundary conditions from Dirichlet boundary conditions. We show rigorous convergence results for the zero diffusion limit, obtained through the renormalized solution and the Fréchet-Kolmogorov theorem. Moreover, we observed several examples through numerical simulations, which are counterintuitive in terms of the classical diffusion law.

Inverse scattering of Alfvén waves in three dimensional ideal magnetohydrodynamics

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In this talk we focus on the inverse scattering problem of nonlinear Alfvén waves governed by the three dimensional ideal incompressible magnetohydrodynamics (MHD) system. By bridging together geometric methods and weighted energy estimates, we establish a set of scattering isomorphisms to substantially strengthen our previous rigidity results. This answer is consistent with the physical intuition that Alfvén waves behave exactly in the same manner as their scattering fields that are detected by faraway observers. The novelty of the present work is twofold: for one thing, the relationship between Alfvén waves emanating from the plasma and their scattering fields at infinities is explored to the best; for another thing, the null structure inherent in MHD equations is thoroughly examined, especially when we estimate the pressure term.

Global stability of a kind of large solution to evolutionary Faddeev model

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In quantum field theory, Faddeev model is an important theory that models heavy elementary particles by knotted topological solitons. In this talk, we show the global nonlinear stability of geodesic solutions of evolutionary Faddeev model corresponding to maps from the Minkowski space $\mathbb{R}^{1+n}$ to the unit sphere $\mathbb{S}^2$. It can be considered as the stability of nontrivial solutions to Cauchy problem of n-dimensional quasilinear wave equations.

An asymptotic preserving method for the weakly nonlinear Klein-Gordon equation

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In this paper, we consider the nonlinear Klein-Gordon equation (NKGE) with weak cubic nonlinearity, i.e., the nonlinearity strength is characterized by ε^2 and a small parameter $\varepsilon \in (0,1)$. Firstly, we apply multi-scale expansion for the weakly NKGE and obtain the first-order approximation term, i.e., $\widetilde{u} = Ae^{i(kx-\omega t)} + A^* e^{-i(kx-\omega t)} + O(\varepsilon)$, where the envelope function A satisfies the nonlinear Gross-Pitaevskii equation. Meanwhile, we make an error estimate on the approximate solution for the long time, i.e., $t \in [0, T/\varepsilon^2]$. Secondly, based on this first-order approximation term, we construct an asymptotic preserving method for the weakly NKGE. Finally, Numerical results are provided to show that (i) the resulting approximation of our asymptotic preserving method can be computed in less time than using methods to solve the weakly NKGE directly instead, (ii) it can conserve the error estimate even for large space and time step sizes, (iii) it is asymptotic preserving.

Dissipative Hyperbolic Mean Curvature Flow

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Hyperbolic curvature flow is a leading topic between nonlinear hyperbolic partial differential equation theory and differential geometry, physics, biomedicine, image processing and so on. In particular, there are practical examples in the theory of Minkowski space-time relativistic strings (or membranes), visual simulation of thin films and soap bubbles, biological tissue growth models, image processing and so on. In this talk, I will describe some recent study on the motion of plane curves under dissipative hyperbolic mean curvature flow. We will talk about two nonlinear evolution equations arising from geometry and bone tissue growth mathematical model, respectively. Some of the discoveries that have been done about them, and some open problems are given.

Parallel Session G1 (SF)

PhD Student Forum
East Middle Hall-Building 1-301,
14:00-18:05

Formation of shifted shock for the 3D compressible Euler equations with damping

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In this talk, we show the shock formation of the solutions to the 3-dimensional (3D) compressible isentropic and irrotational Euler equations with damping for the initial short pulse data. Due to the damping effect, the largeness of the initial data is necessary for the shock formation, and we will work on the class of large data (in the energy sense). Similar to the undamped case, the formation of shock is characterized by the collapse of the characteristic hypersurfaces and the vanishing of the inverse foliation density function μ, at which the first derivatives of the velocity and the density blow up. However, the damping effect changes the asymptotic behavior of the inverse foliation density function μ and then shifts the time of shock formation compared with the undamped case.

On the null controllability of damped nonlinear wave equation

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In this paper, we study the null controllability of nonlinear wave systems. Firstly, by combing Gerlerkin method and a fixed point theorem, we give the null controllability for semi-linear wave equation with nonlinear function dependents on velocities under the geometric control condition. Secondly, based on a new iterative method, we prove the null controllability for a class of quasi-linear wave system constructively. Finally, we give a control result for a class of fully nonlinear wave system as application.

Nonlinear wave interactions in a two-dimensional Riemann problem for a thin-film model of a perfectly soluble anti-surfactant solution

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In mathematical modelling and computational science, analyzing hyperbolic systems of conservation laws in multi-dimension is a crucial and challenging field. It provides invaluable insights into the behaviour of various physical phenomena, from fluid dynamics and electromagnetism to traffic flow and financial markets. Their importance arises from their ability to describe wave propagation such as shocks, rarefaction, and contact discontinuities; even the non-classical delta shock wave, making them indispensable in understanding complex real-world systems.

We analyze both analytically and numerically the construction of nonlinear wave solutions for a reduced hyperbolic model that describes the thin film flow of a perfectly soluble anti-surfactant solution in a three-constant two-dimensional Riemann problem. Moreover, we solve the problem without the limitation that each jump of the initial data emanates exactly one planar elementary wave. We have successfully obtained ten topologically distinct solutions using the generalized characteristic analysis. Our analysis explores the intricate interaction between classical and non-classical waves. Furthermore, we thoroughly compare these analytical solutions with the results obtained using the second-order Local Lax Friedrichs scheme in our numerical simulations, which validates our results.

Mathematical Challenges in Bridge Cable Force Evaluation: Coupling Problems in String Vibration Equations with Elastic Supports

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This study focuses on a critical aspect of bridge engineering—the evaluation of cable forces, paying particular attention to how dynamic parameters change when the cables are internally constrained by elastic supports. These parameters are important for the safety and stability of bridges. The practical problem introduces a novel mathematical challenge: how to effectively address coupling problems in string vibration equations with one or multiple elastic supports, which remains a theoretical issue not fully solved in engineering. To tackle this, it is necessary to first establish an appropriate mathematical model and accurately define initial-boundary value problems. We then formulate the well-posedness of the solutions using both classical and weak solution approaches, supplementing the existing numerical results available in engineering. Meanwhile, we attempt to use a popular numerical simulation known as PINNs (Physics-Informed Neural Networks) instead of the traditional FEM (Finite Element Method) in engineering. Consequently, in contrast to the classical solution method, we demonstrate that for the coupling problems with finite elastic supports, the weak solution method not only improves mathematical modeling efficiency but also simplifies the process of explaining the well-posedness of the solution. This is a joint work with Ke Liu (East China Normal University), Prof. Aifang Qu (Shanghai Normal University), Prof. Qin Xu (Central Research Institute of Building and Construction CO., LTD.MCC Group), Prof. Hairong Yuan (East China Normal University).

Delta Shocks and vacuum states in the Riemann solutions of Chaplygin Euler equations as pressure and magnetic field drop to zero

Priyanka & M. Zafar

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The aim of the present study is to solve the Riemann problem of isentropic magnetogasdynamics equations for a more realistic version of the extended Chaplygin gas

model. The analysis demonstrates that under some special circumstances, delta shock and vacuum appear in the solution, describing the phenomena of concentration and cavitation, respectively. By examining the limiting behavior, it is obtained that solutions coincide with corresponding Riemann solutions of the transport equations when both the magnetic field and pressure drop to zero.

This paper has also been published in the Journal of Mathematical Physics

Publication Detail: Priyanka and M. Zafar. Delta shocks and vacuum states in the Riemann solutions of Chaplygin Euler equations as pressure and magnetic field drop to zero. Journal of Mathematical Physics, 63(12):121505, 2022.

Second order divergence constraint preserving entropy stable finite difference schemes for two-fluid plasma flow equations

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Two-fluid plasma flow equations describe the plasma flow of ions and electrons with different densities, velocities, and pressures. The system of equations has flux consisting of three independent components, one each for ions and electrons flows and a linear Maxwell's equation flux for the electromagnetic fields. The coupling of these components is via source terms.

This article introduces second-order finite difference schemes aimed at maintaining the consistent evolution of divergence constraints on both electric and magnetic fields. The core concept involves developing a numerical solver for Maxwell's equations, utilizing a multidimensional Riemann solver at vertices to ensure discrete divergence constraints. To handle the fluid components, we employ an entropy-stable discretization approach. Our proposed schemes are collocated, boasting second-order accuracy, entropy stability, and ensuring the divergence-free evolution of the magnetic field. Time discretization is accomplished through explicit and Implicit-Explicit (IMEX) schemes. To demonstrate the accuracy, stability, and divergence constraint-preserving ability of the proposed schemes, we present several test cases in one and two dimensions. We also compare numerical results with schemes with no divergence cleaning and perfectly hyperbolic Maxwell (PHM) equations based divergence cleaning schemes for Maxwell's equations.

It's a joint work with Deepak Bhoriya, Harish Kumar, Praveen Chandrashekar, and Dinshaw S. Balsara.

Viscous Destabilization of Large Shock Profiles of Conservation Laws

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I will discuss stability properties for viscous perturbations of conservation laws. The recent theory of a-contraction with shifts yields L^2-type stability estimates for small-amplitude viscous shock profiles. These results are similar to previous work on L^2-type stability for inviscid shock profiles. In some sense, this shows that the L^2-stability theory of the viscous model and the inviscid model are similar for small-amplitude shocks. I will present results showing that for large-amplitude shocks, the situation is markedly different between the two. This is joint work with Paul Blochas and Alexis Vasseur.

Well-posedness of the two-dimensional isentropic compressible Navier-Stokes equations with degenerate viscosities and far field vacuum

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In this paper, we delve into the Cauchy problem pertaining to the two-dimensional (2-D) isentropic compressible Navier-Stokes equations. When the viscosity coefficients are expressed as a constant multiple of the density's power (ρ^δ with $0 < \delta < 1$), we undertake an analysis of the system's nonlinear structure. As a result, by introducing novel variables and initial compatibility conditions, we identify a subset of initial data that permits a local regular solution with far field vacuum and finite energy within certain inhomogeneous Sobolev spaces. Furthermore, our findings reveal that it is not feasible to obtain a global regular solution where the L^∞ norm of u diminishes to zero as time t approaches infinity.

A contact discontinuity resolving Boltzmann scheme based on peculiar velocity

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In the quest for developing robust and accurate numerical methods for simulating compressible flows, kinetic schemes or Boltzmann schemes represent elegant alternatives to traditional Riemann solvers. Kinetic schemes developed in previous decades were known to be entropy stable and robust but yet unable to accurately recognize shock waves and/or contact discontinuities. These features were pursued recently, addressing in particular exact discontinuity capturing [1], [2] and entropy conservation/stability [3] in this framework. In this work, we present yet another Boltzmann scheme which, while retaining the desired physical features and robustness introduced in an earlier work [4], is capable of resolving steady contact discontinuities exactly without numerical diffusion.

In the original work [4], the separation of fluid and peculiar velocities (and separate the upwinding based on each of them), is a desired feature leading to convection-pressure splitting at the macroscopic level. With an eye on facilitating the introduction of genuinely multidimensional models, we retain this foundation. However, the original scheme is overly diffusive and cannot recognize any discontinuities. The consistency analysis reveals that the peculiar velocity based diffusion coefficient does not vanish as the Mach number goes to zero. This is the primary reason for the diffusive prediction of the steady contact discontinuities and slip surfaces by the peculiar velocity based upwind Boltzmann scheme. We try to overcome this limitation by enforcing the isentropy conditions in the part based on peculiar velocity, thereby leading to the recognition of steady contact discontinuities, taking inspiration from the fact that the contribution of the peculiar velocity part to the macroscopic entropy flux is zero. Preliminary results indicate that the robustness of the original scheme is preserved, while still improving the accuracy of the algorithm, apart from capturing the grid-aligned steady contact discontinuities exactly.

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6 July 2, Tuesday

Parallel Session A2 (CT)

Contributed Talks
East Middle Hall-Building 1-107,
14:00-18:05

Delta shock solution for a generalized zero-pressure gas dynamics system with a forcing term

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In this paper, we show some results of the Riemann problem to the following generalized zero-pressure gas dynamics system with a forcing term

$$\begin{cases} \rho_t + (\rho u^k)_x = 0, \\ (\rho u)_t + (\rho u^{k+1})_x = G(\rho, u), \end{cases} \tag{1}$$

where k is an odd natural number and $G(\rho, u) = -\rho u$ or $G(\rho, u) = \alpha\rho$, α a nonzero constant friction coefficient. When $k = 1$ and $G(\rho, u) \equiv 0$, the previous system is called *zero-pressure gas dynamics system* and it has been extensively studied due to its importance in applications. The zero-pressure gas dynamics system is referred to as the adhesion particle dynamics system to describe the motion process of free particles sticking under collision in low temperature and the formation of large-scale structures in the universe [1,2,3,4]. In this talk, we show Riemann solutions to the system (1) using the vanishing viscosity method by means of an auxiliary viscous system with a similarity variable.

[1] F. Bouchut, On zero pressure gas dynamics. In: Advances in Kinetic Theory and Computing. Series on Advances in Mathematics for Applied Sciences, vol. 22, pp. 171–190. World Scientific, Singapore (1994).

[2] Y. Brenier and E. Grenier, Sticky particles and scalar conservation laws. SIAM J. Numer. Anal. 35(6), 2317–2328 (1998).

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Equivalence of entropy solutions and gradient flows for the 1D pressureless Euler–Poisson system

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We consider the pressureless Euler–Poisson system for initial data $\bar{\rho} \in \mathcal{P}_2(\mathbb{R})$, a probability measure with bounded second moment, and $\bar{v} \in L^2(\mathbb{R}, \bar{\rho})$. For unknowns ρ_t and v_t this reads

$$\begin{aligned} \partial_t \rho_t + \partial_x(\rho_t v_t) &= 0, \\ \partial_t(\rho_t v_t) + \partial_x(\rho_t v_t^2) &= -\alpha \rho_t \partial_x \phi, \end{aligned} \tag{2}$$

where $\alpha \in \mathbb{R}$, and ϕ satisfies the Poisson equation in distributions, i.e., $\partial_{xx}\phi = \rho_t$. Here $\alpha > 0$ corresponds to an attractive force, which could describe self-gravitating matter, while $\alpha < 0$ yields a repulsive force, for which (2) could serve as a model for plasma. This system has attracted much attention, and in [4] the authors derived a scalar conservation law for $M_t(x) = \rho_t((-\infty, x])$ with a time-dependent flux function $\mathcal{U}(t, \cdot)$. That is,

$$\partial_t M_t + \partial_x \mathcal{U}(t, M_t) = 0, \tag{3}$$

for which the authors prove existence and uniqueness of the corresponding entropy solutions.

For $\alpha = 0$, (2) reduces to the pressureless Euler system, studied in [3] using L^2-gradient flows, which are linked to a gradient flow for the Wasserstein metric. This idea was extended to pressureless Euler systems with self-interacting force distributions in [2], for which (2) is a special case. The associated L^2-gradient flow X_t satisfies a differential inclusion involving the subdifferential $\partial I_{\mathcal{K}}$ of the indicator function $I_{\mathcal{K}}$ of a convex subset $\mathcal{K}$, and reads

$$\dot{X}_t + \partial I_{\mathcal{K}}(X_t) \ni U_t, \tag{4}$$

for a function U_t. There is then a unique solution evolving according to the minimal element of the subdifferential.

Both (3) and (4) provide distributional solutions to (2), which for $\alpha \geq 0$ are globally sticky. We show that these notions of solutions are equivalent; in particular, the Oleĭnik E condition for (3) turns out to be equivalent to the minimal selection criterion for (4). This can be seen as a second-order generalization of the results in [1], where an analogous equivalence is shown for a nonlocal interaction equation with attractive or repulsive Poisson potential.

Based on joint work with José A. Carrillo (University of Oxford).

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On the isentropic van der Waals gas flux perturbed Riemann solution for Cargo-LeRoux model

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In this research, we study the pressureless Cargo-LeRoux model of conservation laws, which is modelled from the one-dimensional constant gravity Euler equations. Introducing flux perturbation of a van der Waals isentropic gas equation of state, the exact solution of Riemann problem is derived and establish the existence and uniqueness of the Riemann solution globally. Finally, the influence of van der Waals excluded volume on the physical quantities is illustrated graphically using MATLAB software.

Time-asymptotic stability of wave patterns to viscous conservation laws

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The talk is concerned with our recent developments on the time-asymptotic stability of wave patterns to the viscous conservation law, based on the $a-$contraction theory and the time-dependent shift techniques. Three main topics will be involved: the time-asymptotic stability of generic Riemann solutions to one-dimensional both barotropic Navier-Stokes equations and full Navier-Stokes equations, the stability of planar wave patterns to the multi-dimensional compressible Navier-Stokes equations and the stability of composite wave of degenerate Oleinik shock and rarefaction wave to a one-dimensional scalar non-convex conservation law.

Solving Riemann Problems with a Topological Tool

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We have developed a novel approach aimed at addressing the intricate challenge of resolving Riemann problems within systems governed by hyperbolic conservation laws. Central to our methodology is the utilization of a topological framework, achieved through the construction of a meticulously designed three-dimensional manifold. This manifold is specifically crafted to facilitate the identification and characterization of solutions to Riemann initial-value problems. In this talk, we will demonstrate the application of our innovative tool to a specific system governed by two conservation laws featuring quadratic flux functions with an elliptic region.

Termed the "wave manifold," our constructed manifold encapsulates points representing potential shock waves, providing a comprehensive platform for exploring the dynamics of hyperbolic systems. Through rigorous calculations and graphical representations, we will delineate the significant curves and surfaces within this wave manifold, offering valuable insights into the underlying structures of the system.

Characteristics, which represent natural surfaces defining rarefactions, and sonic surfaces, the geometrical loci where shocks attain an extremum, play a crucial role in subdividing the wave manifold into distinct regions according to shock wave admissibility. Within this framework, we will disentangle rarefaction, shock, and composite curves, collectively converging to form wave curves, illuminating the intricate interplay of different wave phenomena.

To address Riemann problems within this manifold, we introduce a novel methodology involving the formation and intersection of intermediate surfaces associated with wave curves. This innovative approach offers a robust means of solving Riemann problems, as demonstrated through illustrative examples of the Riemann solution showcased within the wave manifold.

Our research represents a significant advancement in the field of hyperbolic conservation laws. By leveraging the wave manifold and its associated methodologies, we provide a comprehensive framework for understanding and solving Riemann problems, thereby contributing to the broader knowledge base in this area of study. The theory and examples are illustrated in [1].

This work is a joint effort with esteemed colleagues: Cesar S. Eschenazi (UFMG - Brazil), Marlon M. López-Flores (Puc-Rio), Dan Marchesin (IMPA - Brazil), Carlos Frederico Palmeira (Puc-Rio), and Bradley Plohr (Los Alamos - USA). Together, we have collaborated to push the boundaries of research in hyperbolic conservation laws.

Keywords: Riemann problems, Hyperbolic conservation laws, Quadratic flux functions, Topological frameworks, Wave manifold, Bifurcation Structures.

[1] C.S. Eschenazi, W.J. Lambert, M.M. Lopez-Flores, D. Marchesin and C.F.B. Palmeira, *Solving Riemann Problems with a Topological Tool (Extended version).* Available[online]: `https://arxiv.org/abs/2312.17377`, (2023).

Riemann problem and wave interactions in two-layer blood flow through artery

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In this talk, we consider the Riemann problem for a 3×3 hyperbolic system of partial differential equations (PDEs) governing two-layer blood flows through an artery. The characteristics of Riemann invariants lead us to employ a projection of the elementary waves onto the phase plane. Further, we use the properties of these elementary waves to establish the solution of the associated Riemann problem in this phase plane. Moreover, we provide a necessary and sufficient condition on initial data for the existence of a vacuum state. Finally, we discuss all possible cases of wave interaction in this phase plane.

On a general existence result for one-dimensional Hughes model

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The Hughes model is one of several PDE models for the evacuation of human crowds, a phenomenon which has gained significant attention in recent years. The model considers an evacuation domain with several exits, where pedestrians choose the exit they perceive to be closest, while avoiding crowded regions. The possibility of pedestrians changing their preferred exit leads to interesting technical challenges, as such turning points correspond to discontuities in the velocity field. I will present the one-dimensional variant of the model, which takes the form of a scalar conservation law with a discontinuous flux function. I discuss an existence result which holds for L^∞ initial data and general non-linear running costs.

[1] D. Amadori, B. Andreianov, M. Di Francesco, S. Fagioli, T. Girard, P. Goatin, P. Markowich, J.-F. Pietschmann, M. D. Rosini, G. Russo, G. Stivaletta and M.-T. Wolfram, The mathematical theory of Hughes' model: a survey of results, preprint, 2023, arXiv:2305.10076.

Asymptotic behavior of hypersonic potential flow past a large curved wedge

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In this talk, we will discuss the global existence of piecewise smooth solutions to the problem of hypersonic potential flow past a large curved wedge with a given density.

Attention will be focused on the asymptotic behavior of the flow at infinity, as well as the convergence of the solution as the Mach number of the incoming flow approaches infinity. Slip boundary conditions are assumed on the surface of the wedge.

Simple waves and double waves of multi-dimensional hyperbolic equations for conservation laws

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In this talk, we are concerned with simple waves and double waves of flow patterns to multi-dimensional hyperbolic equations for conservation laws. The flow region of simple waves is covered by a family of one-parameter surfaces, on each of which flow is constant state. The double waves are a type of flow whose flow region is covered by a two-parametric family of independent curves, along each of which flow is also constant state. The two-dimensional scaler hyperbolic equation for conservation law and the three-dimensional isentropic irrotational pseudo-steady flow are studied in some more detail.

Jointed with Prof. G.Lai, Prof.Y.X.Cai, and Miss Da.

Parallel Session B2 (CT)

Contributed Talks
East Middle Hall-Building 1-109,
14:00-18:05

Unconditional stability of equilibria in thermally driven compressible fluids

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We show that small perturbations of the spatially homogeneous equilibrium of a thermally driven compressible viscous fluid are globally stable. Specifically, any weak solution of the evolutionary Navier–Stokes–Fourier system driven by thermal convection converges to an equilibrium as time goes to infinity. The main difficulty to overcome is the fact the problem does not admit any obvious Lyapunov function. The result applies, in particular, to the Rayleigh–Bénard convection problem.

Global decay of perturbations of equilibrium states of one-dimensional compressible viscous fluids of Korteweg type

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In this work we study the dissipative structure of the system of equations describing a compressible viscous fluid of Korteweg type, for both the Navier-Stokes-Korteweg (NSK) system [3] and the Navier-Stokes-Fourier-Korteweg (NSFK) system [4] that correspond to the isothermal and non-isothermal case, respectively. The systems are analyzed under the framework of Humpherys's equivalence theorem [1] for higher-order systems, which is, indeed, a generalization of the classical one by Shizuta and Kawashima [5] for second-order systems. We show that the systems are symmetrizable in the symbol, a notion of symmetrizability introduced by Humpherys [1] that generalizes the classical one of Friedrichs. The (symbol) symmetric systems are shown to satisfy the (generalized) genuine coupling condition, implying the existence of a (symbol) compensating matrix function. We construct appropriate compensating matrix functions allowing us to obtain linear decays which underlie a mechanism of regularity-gain type [2]. This linear dissipative structure implies the global decay of perturbations to constant equilibrium states as solutions to the full nonlinear systems, via a standard continuation argument. For this final step, although for the NSK system this is done easily by working in Lagrangian variables, for the NSFK system a nonlinear change of perturbed variables, which makes use of the entropy of the fluid and takes into consideration that the state variables also depend on density gradients, is introduced.

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[4] R. G. Plaza and J. M. Valdovinos, *Global decay of perturbations of equilibrium states for one-dimensional heat conducting compressible fluids of Korteweg type*, Arxiv preprint: 2307.16300. Submitted to J. Diff. Equ.

[5] Y. Shizuta and S. Kawashima, *Systems of equations of hyperbolic-parabolic type with applications to the discrete Boltzmann equation*, Hokkaido Math. J. **14** (1985), no. 2, pp. 249–275.

Asymptotic behavior of solution for the compressible Navier-Stokes equations

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We first show the nonlinear stability result of the planar viscous shock up to a time-dependent shift for the three-dimensional compressible Navier-Stokes equations under the generic perturbations, in particular, without zero mass conditions. Next, we are concerned with the vanishing dissipation limiting problem of one-dimensional non-isentropic Navier-Stokes equations with shock data.

Long Time Existence for Slightly Compressible Navier-Stokes Equations in Bounded Domains

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We will present our recent studies concerning the long time existence for slightly compressible Navier-Stokes equations in bounded domains with slip or non-slip boundary conditions. No smallness restrictions are imposed on the initial data and the time interval. If the limiting system admits a reasonably smooth solution on a certain period, we verify that the corresponding compressible system admits the smooth solution on the same duration as well, provided the Mach number is small enough. Moreover, the solutions of compressible system converge uniformly to that of the incompressible one as Mach number tends to zero. We apply the global geometric tools introduced by Chrisrodoulou-Lindblad to obtain higher order estimates for the solution near the boundary

when dealing with the case of non-slip boundary conditions. This is joint work with professor Qiangchang Ju and Dr. Xinyu Fan.

Large amplitude traveling waves for the non-resistive MHD system

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The goal of this talk is to discuss the existence of large amplitude traveling waves of the two-dimensional non-resistive Magnetohydrodynamics (MHD) system with a traveling wave external force. More precisely, we assume that the force is a smooth bi-periodic traveling wave propagating in the direction $\omega = (\omega_1, \omega_2) \in \mathbb{R}^2$, with large amplitude of order $O(\lambda^{1^+})$ and with large velocity speed $\lambda\omega$. Then, for most values of ω and for $\lambda \gg 1$ large enough, we construct bi-periodic traveling wave solutions of arbitrarily large amplitude. Due to the presence of small divisors, the proof is based on a nonlinear Nash-Moser scheme adapted to construct nonlinear waves of large amplitude. The main difficulty is that the linearized equation at any approximate solution is an unbounded perturbation of large size of a diagonal operator and hence the problem is not perturbative. The invertibility of the linearized operator is then performed by using tools from micro-local analysis and normal forms together with a sharp analysis of high and low frequency regimes with respect to the large parameter λ.

Some recent progress on blowup criteria for compressible Navier-Stokes equations

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In this talk, we will introduce some recent progress on blowup criteria in terms of concentration of density and absolute temperature for viscous, compressible, and heat-conducting flow with vaccum.

Well-posedness and Incompressible limit of Current-Vortex Sheets in Ideal Compressible MHD

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Abstract: The current-vortex sheet, as one of the characteristic contact discontinuities in ideal compressible MHD, is described by a free-interface problem for two-phase MHD flows with magnetic fields tangential to the interface. We prove local well-posedness and incompressible limit for current-vortex sheets with or without surface tension. The key observation is a hidden structure of Lorentz force in vorticity analysis which motivates us to use certain anisotropic Sobolev spaces instead of standard Sobolev spaces.

Some results on rough solutions of 3D compressible Euler equations

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For the Cauchy problem of irrotational Euler equations and incompressible Euler equations, the sharp regularity has been obtained for controlling the local existence of solutions. However, the sharp regularity problem for compressible Euler equations with non-trivial vorticity and entropy remains open. The best known 3D result is proved by Qian Wang in 2019. In this talk we present an approach leading well-posedness results for the 3D compressible Euler equations with relaxed assumptions on the regularity of the vorticity. This is a joint work with Lars Andersson.

Outflow/Inflow Problem for Two-Phase Flow

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In this talk, we present recent investigation on inflow/outflow problem for the two-phase flow, which consists of two compressible Navier-Stokes equations coupled with each other through the drag force relaxation mechanisms and is derived by the Chapman-Enskog expansion from the Vlasov-Navier-Stokes equations for the mixed fluid-particle motion. The existence of the unique steady-state is shown respectively corresponding to the supersonic, sonic, or subsonic state at the far field. The nonlinear stability and long time convergence rates are also established. Meanwhile, the comparison of the two-phase flow with single-phase compressible viscous equations and the drift-flux two-phase flow models are made. This is joint with Prof. Hai-Liang Li (CNU).

Parallel Session C2 (CT)

Contributed Talks
East Middle Hall-Building 2-203,
14:00-17:40

Macroscopic estimate of Boltzmann equation with mixed boundary

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In this talk, I will discuss the Boltzmann equation in a bounded domain. In the first part, we consider a mixed boundary condition where a portion of the boundary follows the pure specular boundary condition, while the remaining portion follows the pure diffuse boundary condition. We investigate the dynamical stability of the nonlinear problem by employing the L^2-L^∞ framework within a special geometry. In the second part, we consider the pure specular boundary condition. We establish an L^6 bound for the macroscopic component of the linear problem. The L^6 estimate is crucial in the future study of the hydrodynamic limit under the specular boundary condition. These are joint work with Renjun Duan and Chanwoo Kim.

On the quadratic stability of asymmetric Hermite basis and application to plasma physics

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We analyze the reasons why the asymmetric Hermite basis functions used for the discretization of linear transport generate dynamical systems which are instable. The main reason is that the finite truncation of the infinite moment linear system looses its anti-symmetry property with respect to the Gramm-Schmidt matrix constituted with the quadratic scalar products of two asymmetric Hermite basis. Then we propose an original closed formula for these scalar products. It makes possible the construction of two simple modifications of the linear systems which recover the anti-symmetry property. By construction the new methods are quadratically stable with respect to the natural L2 norm. We explain how to generalize to other transport equations encountered in numeircal plasma physics. Basic numerical tests illustrate the unconditional stability properties of our algorithms.

Two double-parameters limit problems on the Vlasov-Poisson-Landau system

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The Vlasov-Poisson-Landau (VPL) and Euler-Poisson (EP) systems are both fundamental models at the kinetic and fluid levels, respectively, describing dynamics of plasmas under the self-consistent electric interactions. It is well known that the formal Hilbert expansion of VPL gives the EP from which one further can either derive the quasi-neutral Euler system by letting the Debye length tend to zero or obtain the KdV equations under the Gardner-Morikawa transformation. In this talk I present recent results on such two kinds of double-parameters limit problems directly from the VPL system in the corresponding appropriate scalings. We treat the physically most important Coulomb interaction potentials. Our approach is to construct the energy functional and energy dissipation functional via the macro-micro decomposition capturing the singularity of two fluid limit problems with respect to two small parameters: dissipation strength vs dispersion strength. Joint with Dongcheng Yang and Hongjun Yu.

Parameter Reconstruction in Kinetic Equations: an Inverse Problem for Chemotaxis

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On the mesoscopic level, motion of individual particles can be modeled by a kinetic transport equation for the population density $f(t,x,v)$ as a function of time t, space x and velocity $v \in V$. A relaxation term on the right hand side accounts for scattering due to self-induced velocity changes and typically involves a parameter $K(x,v,v')$ encoding the probability of changing from velocity v' to v at location x:

$$\partial_t f(t,x,v) + v \cdot \nabla_x f(t,x,v)$$
$$= \int_V K(x,v,v')f(t,x,v') - K(x,v',v)f(t,x,v) \, \mathrm{d}v'.$$

This hyperbolic model is for instance widely used to model bacterial motion, that might be influenced by an external stimulus (e.g. a chemical gradient), in which case it is termed chemotaxis.

We study the inverse parameter reconstruction problem whose aim is to recover the scattering parameter K and that has to be solved when fitting the model to a real situation. With the chemotaxis application in mind, we restrict ourselves to macroscopic, i.e. velocity averaged data $\rho = \int f dv$ as a basis of our reconstruction. This introduces additional difficulties, given that the parameter $K(x,v,v')$, that we want to reconstruct, depends on the incoming and outgoing velocity. This can be leveraged by the use of short time interior domain data. In this way, we can establish theoretical existence and uniqueness of the reconstruction, study its macroscopic limiting behavior and numerically conduct the inversion under suitable data generating experimental designs.

This work based on a collaboration with Christian Klingenberg (Würzburg, Germany), Qin Li (Madison, Wisc., USA) and Min Tang (Shanghai, China) and has partially been published in the following articles:

- Hellmuth, Klingenberg, Li, Tang,
  Numerical reconstruction of chemotaxis transition kernel,
  submitted (2023)
- Hellmuth, Klingenberg, Li, Tang,
  Kinetic chemotaxis tumbling kernel determined from macroscopic quantities,
  SIAM J. Math. Anal., 56 (2024), pp. 568-587
- Hellmuth, Klingenberg, Li, Tang,
  Multiscale convergence of the inverse problem for chemotaxis in the Bayesian setting,
  Computation 9(11) (2021)

Small inertia limit for coupled kinetic swarming models

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We investigate various versions of multi-dimensional systems involving many species, modeling aggregation phenomena through nonlocal interaction terms. We establish a rigorous connection between kinetic and macroscopic descriptions by considering the small-inertia limit at the kinetic level. The results are proven either under smoothness assumptions on all interaction kernels or under singular assumptions for self-interaction potentials. Utilizing different techniques in the two cases, we prove the existence of a solution to the kinetic system, provide uniform estimates with respect to the inertia parameter, and show convergence towards the corresponding macroscopic system as the inertia approaches zero. This is a joint work with Young-Pil Choi and Simone Fagioli.

Polynomial tail solutions of the Boltzmann equation in the compressible Euler limit

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We develop the method of macro-micro decomposition for the Boltzmann equation for allowing the microscopic component to exhibit only the polynomial tail in large velocities. Specifically, we focus on the Cauchy problem on the non-cutoff Boltzmann equation in the fluid dynamical limit to the compressible Euler system as the Knudsen number tends to zero. Via the macro-micro decomposition, up to any finite time we construct the Boltzmann solutions around a local Maxwellian whose fluid quantities are the small-amplitude classical solutions to the corresponding full compressible Euler system around constant states. We design a new energy functional which can capture the convergence rate in Knudsen number and allow the microscopic part of solutions to decay in large velocities polynomially.

Mathematical Analysis of Plasma Sheath Formation

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The purpose of this talk is to mathematically investigate the formation of a plasma sheath near the surface of walls immersed in a plasma. Langmuir [5] observed that in order to form a sheath, positive ions must enter the sheath region with a sufficiently large flow velocity. Bohm [3] proposed the original Bohm criterion using the Euler–Poisson system. The Bohm criterion states that the flow velocity of positive ions at the plasma edge must exceed the ion acoustic speed. There have been a large amount of mathematical results investigating the sheath formation by using the Euler–Poisson system ever since. Beside those studies using the hydrodynamic model, it is also of important to analyze the sheath formation from a kinetic point of view. Boyd–Thompson [4] proposed a kinetic Bohm criterion $\int_{\mathbb{R}^3} \xi_1^{-2} F_\infty(\xi) d\xi \leq 1$, where ξ_1 and F_∞ denote the normal velocity to the boundary and the velocity distribution of positive ions at a sheath edge, respectively. Then Riemann [5] pointed out (although without a rigorous proof) that the kinetic Bohm criterion is a necessary condition for the stationary Vlasov–Poisson system to be solvable. For the studies with rigorous proofs, Badsi–Campos Pinto–Després [2] proved the existence of stationary solutions over an interval by assuming the condition $\int_{\mathbb{R}^3} \xi_1^{-2} F_\infty(\xi) d\xi < 1$. The linear stability of the stationary solutions was also studied in [1]. Recently, the paper [7] analyzed the solvability of the stationary problem of the Vlasov–Poisson system over a half-space, and clarified in all possible cases whether or not there is a stationary solution. It was concluded that the Bohm criterion is necessary but not sufficient for the solvability.

In this talk, we study the nonlinear stability and instability of the stationary solutions of the Vlasov–Poisson system in some weighted Sobolev space in certain situations. The main difficulty of our proof of the stability lies on the fact that the Vlasov–Poisson system has no dissipation. On the other hand, the completely absorbing boundary condition has a part of outflow type, which is a kind of dissipation. To make use of this dispersion structure, we employ a weighted Sobolev space and track the location of the support of the velocity distribution F. It is concluded that the support of the initial data F_0 is a major factor leading to stability/instability. Indeed, the ξ-support of F being constrained to $\{\xi \in \mathbb{R}^3 \,|\, \xi_1 < 0\}$ helps for stability, where particles with the negative normal velocity $\xi_1 < 0$ go to the boundary. A nonempty intersection of the ξ-support and $\{\xi \in \mathbb{R}^3 \,|\, \xi_1 > 0\}$ helps for instability.

This talk is based on a joint work with Professor M. Takayama (Keio University) and Professor K. Z. Zhang (Northeastern University).

[1] M. Badsi, *Linear electron stability for a bi-kinetic sheath model*, J. Math. Anal. Appl. **453** (2017), 954–972.

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[5] I. Langmuir, *The interaction of electron and positive ion space charges in cathode sheaths*, Phys. Rev. **33** (1929), 954–989.

[6] K.-U. Riemann, *The Bohm criterion and sheath formation*, J. Phys. D: Appl. Phys. **24** (1991), 493–518.

[7] M. Suzuki and M. Takayama, *The Kinetic and Hydrodynamic Bohm Criterions for Plasma Sheath Formation*, Arch. Ration. Mech. Anal. **247** (2023), 86.

[8] M. Suzuki, M. Takayama and K. Z. Zhang, *Nonlinear Stability and Instability of Plasma Boundary Layers*, arXiv:2208.07326.

Smoothness property of hypoelliptic kinetic equations near boundaries

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The boundary regularization effect for hypoelliptic kinetic equations is limited. The solution within a bounded domain exhibits at most Hölder continuity around the singular set of the boundary. We will discuss some recent results on the hypoelliptic regularity and explain smoothness properties of solutions under the inflow or reflection boundary conditions in certain cases.

Parallel Session D2 (CT)

Contributed Talks
East Middle Hall-Building 2-303,
14:00-18:05

Local Characteristic Decomposition Based Path-Conservative A-WENO Schemes

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We propose a new less diffusive fifth-order flux globalization based path-conservative alternative weighted essentially non-oscillatory (A-WENO) scheme for general nonconservative systems. In order to reduce the diffusion in the recently proposed flux globalization based path-conservative A-WENO scheme, we use a local characteristic decomposition procedure to develop a new class of numerical flux. We apply the developed schemes to the one- and two-dimensional γ-based multifluid systems to illustrate the performance on a variety of examples. The obtained numerical results clearly demonstrate that the proposed new schemes outperform the original path-conservative A-WENO schemes.

Temporal-Spatial Coupling Finite Volume Scheme for the Kapila Model of Compressible Multiphase Flows

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A second-order accurate and robust numerical scheme is developed for the Kapila model to simulate compressible multiphase flows. The scheme is formulated within the temporal-spatial coupling framework with the generalized Riemann problem (GRP) solver applied as the cornerstone. The use of the GRP solver enhances the capability of the resulting scheme to handle the stiffness of the Kapila model in two ways. Firstly, in addition to Riemann solutions, the time derivatives of flow variables at cell interfaces are obtained by the GRP solver. The coupled values, i.e. Riemann solutions and time derivatives, lead to a straightforward approximation to the velocity divergence at the next time level, enabling a semi-implicit time discretization to the volume fraction equation. Secondly, the use of time derivatives enables numerical fluxes to comprehensively account for the effect of the source term, which includes interactions between phases. The robustness of the resulting numerical scheme is therefore further improved. In addition, a test case with a nonlinear smooth solution is designed to demonstrate the numerical accuracy.

Bound- and Positivity-preserving Affine-invariant alternative WENO Scheme for the

Five-equation Model of Two-medium Flows

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Numerical study on compressible two-medium flows has been a hot issue in recent decades due to broad applications in gas bubble dynamics, underwater explosion, inertial confinement fusion, and so on. In this study, we present a fifth order finite difference path-conservative alternative WENO scheme with central-upwind numerical flux (PCCU-AWENO) for the five-equation transport model (Allaire, JCP, 2002) of two-medium flows with the stiffened gas equation of state. We propose a uniformly high order flux-based bound- and positivity-preserving (BP-P) limiters for the scheme, while preserving the equilibrium solutions simultaneously. Once the BP-P limiters are used, oscillations may be generated at material interface. To this end, we introduce the affine-invariant WENO interpolations, which take care of structures, especially in small scales. In addition, we will systematically derive the BP-P CFL conditions. We will explain how to choose a proper family of paths for the equation with non-conservative product to improve BP-P CFL conditions. If time permits, we also introduce BP-P PCCU scheme in the framework of finite volume methods.

Reinforcement Learning Based Shock Capturing for High-Order Methods

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A characteristic property of non-linear hyperbolic conservation laws is the breakdown of classical solutions and the occurrence of discontinuities. First-order finite volume (FV) schemes in combination with a monotone numerical flux function turned out to be well suited to solve this type equations. However, in practical applications, discontinuities occur only on sets of small measure while for the by far larger part of the domain the solution varies smoothly. Especially for two- and three-dimensional problems, first-order schemes require a massive grid resolution to be accurate, and high-order methods are favorable in general. However, high-order methods are prone to oscillations at strong discontinuities. Hence, non-linear methods based on TVD limiting strategies or essentially non-oscillatory reconstruction schemes of (W)ENO-type have been proposed. However, a major drawback of these schemes is the optimal choice of the limiter function with respect to stability and accuracy. Sophisticated a priori slope limiters have been proposed that allow the limiter to be tuned to the particular test case and, in general, to strike a balance between robustness and accuracy of the results. An alternative strategy to the discussed a priori limiters is the MOOD approach, where the validity of the solution is examined in an a posteriori fashion based on some predefined admissibility criteria. If any of the defined criteria is violated, the solution of the corresponding elements has to be recomputed with a more robust scheme. This procedure enables a nearly oscillation-free treatment of discontinuities. Moreover, several researchers proposed data-driven shock detection [1] or capturing methods [2]. However, they have in common that the training was performed in a supervised learning (SL) fashion, i.e., none of the models actually took the response of the numerical scheme to their prediction into account.
In the present talk, an alternative approach presented in [3,4] is discussed which utilizes methods from reinforcement learning (RL), where in contrast to SL, the training process is carried out by allowing the trained model itself to interact with an environment. The aim is to design an a priori limiter which has the MOOD properties of an a posteriori limiter. To this end, the environment is a two-dimensional MUSCL-Hancock scheme and the agent is a neural network. The outline of this talk is as follows. First, a short introduction to RL is given. This is followed by the discussion of our training approach and the corresponding RL framework is presented. Turning to numerical experiments, the applicability and predictive performance of the developed limiter is outlined on the example of the Euler equations in combination with an ideal gas equation of state. Finally, the extension of the proposed framework to the high-order discontinuous Galerkin method is discussed. We conclude the talk with a critical examination of the potential as well as the drawbacks of our approach.

[1] D. Ray and J. S. Hesthaven, ”Detecting troubled-cells on two-dimensional unstructured grids using a neural network”, Journal of Computational Physics 397, 108845 (2019).

[2] M. Han Veiga and R. Abgrall, ”Towards a general limiter for systems of conservation laws: 1D scalar and system of equations”, ECCM ECFD VII, Glasgow, 2525–2550 (2018).

[3] A. Schwarz, J. Keim, S. Chiocchetti and A. Beck, ”A Reinforcement Learning Based Slope Limiter for Second-Order Finite-Volume Schemes”, GAMM22 (PAMM22, 2023).

[4] J. Keim, A. Schwarz, S. Chiocchetti, C. Rohde and A. Beck, ”A Reinforcement Learning Based Slope Limiter for Two-Dimensional Finite Volume Schemes”, FVCA X, 2 (2023).

Structure preserving numerical methods for multi-d hyperbolic conservation laws

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The solutions of multi-dimensional conservation laws have many more phenomena than their one-dimensional counterparts. Examples are vortices and also low Mach number flows, where the flow becomes almost incompressible (a solenoidal velocity is of interest only in multi-d.). For linear systems of conservation laws one can identify all those finite volume discretization that resolve these multi-d phenomena also on coarse grids extremely well. In addition they turn out to also exactly preserve discrete stationary solutions. We shall call this class of finite volume methods *stationarity preserving*.

The quest now is to find a non-linear counterpart to such structure preserving numerical finite volume schemes. We define a finite volume scheme for non-linear hyperbolic conservation laws to be *stationarity preserving*, if, when our scheme is applied to the linearization of the conservation laws, the scheme is *stationarity preserving*. Next we shall present a class of finite volume schemes inspired by Phil Roe's Active Flux method. It is applicable to systems of conservation laws (so it is not restricted to the Euler equations) and it is of arbitrary high order. We can show that according to our above definition such schemes are *stationarity preserving*. In numerical experiments these schemes, when applied to the non-linear equation, show the structure preserving properties we seek.

This is joint work with among others with Remi Abgrall, Wasilij Barsukow and Lisa Lechner (a PhD student in Würzburg).

High-order central-upwind schemes for non-conservative hyperbolic systems

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I will discuss how central-upwind schemes, including the recently proposed local characteristic decomposition based central-upwind schemes, can be applied to general nonconservative hyperbolic systems via the flux globalization approach. A high order of accuracy of the presented schemes will be achieved within the finite-difference A-WENO framework. Performance of the new schemes will be illustrated on applications to the nozzle flow system, two-layer shallow water equations, and multicomponent compressible Euler equations.

A priori error estimates of Runge–Kutta discontinuous Galerkin schemes to sommth solutions of fractional conservation laws

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We consider the nonlinear scalar fractional conservation law (FCL)

$$\partial_t u + \partial_x f(u) = -(-\partial_x^2)^{\lambda/2}(u) \qquad \text{in} \quad \mathbb{R}\times(0,T), \quad (5)$$

where $-(-\partial_x^2)^{\lambda/2}$ is the nonlocal fractional Laplace operator with $\lambda \in (0,1)$. The fractional Laplacian can be used to model diffusion processes that are governed by Lèvy processes. Physically motivated applications may range from molecular biology and radiation to hydrodynamics as well as mathematical finance.

It is well known that solutions to FCLs may develop shocks in finite time if the diffusion fails to counterbalance the convection. Accordingly, in the context of numerical methods, low convergence rates have to be expected and have indeed already been proven by Cifani et al. [1]. Contrary to these worst-case estimates, one may be interested in achievable convergence rates when dealing with sufficiently smooth solutions to obtain a more differentiated picture of the numerical performance. This line of reasoning has also been followed in the setting of pure conservation laws $\partial_t u + \partial_x f(u) = 0$, which share some properties with FCLs.

In fact, for smooth solutions to conservation laws, Zhang and Shu [2] derived the a-priori error estimate

$$\|u(t^n) - u_h^n\|_{L^2(\mathbb{R})} \lesssim h^{k+1} + \tau^2, \quad (6)$$

for a second order Runge-Kutta discontinuous Galerkin (RKDG) method by using Taylor expansion and energy estimates, ultimately relying on the Gauss-Radau projection. Note that here and in the following, h is the maximal element width, τ is the time-step, and the numerical solution u_h^n at time t^n is sought in the space $\mathbb{P}^k$ of broken polynomials of degree k

At each time level t^n, Zhang and Shu introduce a projection operator $\mathbb{R}_h$ that actually depends on the exact solution $u^n = u(t^n)$: If $f'(u^n)$ is positive on any given element, interpolation on Radau points including the right boundary point is used on that element. Conversely, if $f'(u^n)$ is negative, interpolation on Radau points including the left boundary point is used. Finally, if $f'(u^n)$ changes sign on some element then the standard L^2 projection is used on that element.

The study of characteristics shows that if $f'(u(x,0)) = 0$ for some $x \in \mathbb{R}$, then $f'(u(x,t)) = 0$ for all $t > 0$ such that on each interval where $f'(u^0)$ is positive (negative/changes sign), the same is true for $f'(u^n)$. In particular, the projection operator $\mathbb{R}_h$ of Zhang and Shu does not depend on time and one has the important approximation property

$$\|\eta_u^{n+1} - \eta_u^n\|_{L^2(\mathbb{R})} \lesssim h^{k+1}\tau, \quad (7)$$

for the projection error $\eta_u^n := \mathbb{R}_h(u^n) - u^n$. In a subsequent Gronwall Lemma, this leads to an error of magnitude h^{k+1}.

In the context of FCLs, the characteristics argument is not viable and thus one must consider a time dependent projection operator $\mathbb{R}_h^n$, in general. As a consequence, we are left with only

$$\|\eta_u^{n+1} - \eta_u^n\|_{L^2(\mathbb{R})} \lesssim h^{k+1}, \quad (8)$$

for times t^n at which $\mathbb{R}_h^{n+1} \neq \mathbb{R}_h^n$, in contrast to (7). Due to summation over all time-steps in the anticipated Gronwall Lemma, the approximation property (8) would yield an error of magnitude $h^{k+1}\tau^{-1}$.

In our work, we introduce a novel upwind projection operator, denoted by Π_h^n, that accommodates a tolerance w.r.t. the mesh width h. By the regularity assumptions on the flux f and the exact solution u we are then able to effectively gauge the number of times N_j for which $\Pi_h^{n+1} \neq \Pi_h^n$ on any element I_j. This ultimately leaves us with improved bounds in the Gronwall argument and consequently in the a priori estimate:

Theorem. *Let $\alpha \geq 3$, $f \in C^{\alpha+1}(\mathbb{R})$ and $u \in C^\alpha([0,T]; H^{k+1}(\mathbb{R}))$ be the exact solution to the FCL (5). Let $u_h^n \in \mathbb{P}^k$, $k \geq 1$, be the numerical solution at time level t^n of our second order in time upwind RKDG scheme with CFL condition $\tau \leq ch$ for a certain $c > 0$ in the case $k = 1$ and $\tau \lesssim h^{4/3}$ for $k \geq 2$. Then we have*

$$\|u^n - u_h^n\|_{L^2(\mathbb{R})} + \left(\sum_{m=0}^{n-1} \tau |u^m - u_h^m|^2_{H^{\lambda/2}(\mathbb{R})}\right)^{1/2} \lesssim h^{k+1-\frac{1}{\alpha}} + h^{k+1-\frac{\lambda}{2}} + \tau^2, \quad (9)$$

for all $n \in \{1, \dots, N\}$.

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High-Resolution Third-Order Hybrid WENO Scheme for Hyperbolic Conservation Laws

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This work presents a high-resolution third-order hybrid finite difference weighted essentially non-oscillatory (WENO) [1,2,3,4]scheme to solve hyperbolic conservation laws. The hybrid algorithm is two-fold. 1. We divide the computational domain into smooth and non-smooth parts based on a troubled cell indicator [5]. This proposed troubled cell indicator is designed based on smoothness indicators of the WENO scheme. It is observed that the smoothness indicator is proportional to wave speed in smooth regions and may violet in the presence of discontinuities. As a result, it has given a freedom to use an upwind scheme in smooth regions and the WENO algorithm in non-smooth regions. 2. It is well known that third-order WENO reconstruction detects discontinuities and critical points as either discontinuities or critical points [6]. To avoid this, a novel and simple switching mechanism between WENO and a linear upwind scheme is introduced. The switching mechanism searches for the smooth extrema in such a way that it implements the linear high-order reconstructed numerical flux if the smooth extreama lies outside the stencil and a WENO procedure is applied to compute numerical flux if the smooth extrema lie inside the stencil. We implement the TVD-RK3 scheme for time integration. The advantage of this new scheme lies in its robustness, efficiency, and high resolution. Several numerical test cases from scalar to system of Euler equations are presented to demonstrate the performance of the proposed hybrid method.

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A new fifth-order hybrid unequal-sized WENO scheme based on trigonometric polynomials for hyperbolic conservation laws

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Weighted essentially non-oscillatory (WENO) scheme is a research hotspot of high-order numerical algorithms for hyperbolic conservation laws, but the calculation costs associated with its local characteristic decomposition, smooth indicators, flux splitting and complex nonlinear weights are relatively high. How to inherit the excellent characteristics of the WENO scheme while reducing its computational costs has always been an open topic. Therefore, we combine the existing unequal-sized WENO (US-WENO) scheme with the linear upwind scheme to construct a hybrid US-WENO (HUS-WENO) scheme. At the same time, in order to make the constructed high-order HUS-WENO scheme easier to solve problems with high-frequency oscillations or large gradient wave phenomena, we use new trigonometric polynomials instead of algebraic polynomials for reconstructing the HUS-WENO (HUS-TWENO) scheme. It is worth mentioning that we utilize the reconstructed trigonometric polynomial on the five-point stencil during the reconstruction process of the US-WENO (US-TWENO) scheme based on the new trigonometric basis functions to design a discontinuous recognizer that does not contain any artificial parameters related to the problems and can automatically, accurately and efficiently identify the troubled cells. Therefore, the constructed HUS-TWENO scheme can greatly improve its computational efficiency while inheriting the excellent characteristics of the US-TWENO scheme. Finally, some benchmark numerical examples are conducted on the classical one-dimensional and two-dimensional Euler equations to verify the good performance of the proposed HUS-TWENO scheme. These numerical experiments show that compared with the US-TWENO scheme, the HUS-TWENO scheme can save about 40% to 70% CPU time; the numerical errors of the HUS-TWENO scheme are smaller than these of the US-TWENO scheme on the same meshes, and the performance of the HUS-TWENO scheme is better than that of the US-TWENO scheme; the discontinuity recognizer proposed can automatically recognize discontinuous structures without adjusting human parameters.

Parallel Session E2 (CT)

Sino-German session on advanced numerical methods for hyperbolic balance laws
East Middle Hall-Building 1-201,
14:00-18:05

Hyperbolic aspects of mixed-dimensional modelling for two-phase flow in fractured porous media

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We consider two-phase flow of two immiscible and incompressible fluids in a porous medium that contains debris-filled fractures. We assume that the aperture of the fractures is small enough to justify the use of discrete-fracture models on the Darcy scale. For this approach, we assume that the fractures can be represented as dimensionally reduced manifolds embedded in the bulk porous medium.
Precisely, we assume that some domain $\mathcal{D} \subset \mathbb{R}^d$, $d \in \{2,3\}$, is partitioned as $\mathcal{D} = D \cup \Gamma$, with the fracture Γ of aperture $\omega : \Gamma \to [0,\infty)$ as a manifold of dimension $d-1$ or $d-2$, and $D \subset \mathbb{R}^d$ being the bulk porous medium domain. If we neglect gravitational and capillary forces, the (wetting) fluid saturation $S = S(\mathbf{x},t) \in [0,1]$, the global pressure $P = P(\mathbf{x},t) \in \mathbb{R}$, and the total velocity $\mathbf{v} = \mathbf{v}(\mathbf{x},t) \in \mathbb{R}^d$ are supposed to satisfy the hyperbolic-elliptic equation

$$\begin{array}{c} S_t + \operatorname{div}\big(\mathbf{v} f(S)\big) = 0, \\ \operatorname{div}(\mathbf{v}) = 0, \quad \mathbf{v} = -\lambda(S)\mathbf{K}\nabla P \end{array} \quad \text{in } D,\, t>0. \qquad (10)$$

In (10), the fractional flux $f = f(S)$, the total mobility $\lambda = \lambda(S)$, and the symmetric, positive definite intrinsic permeability tensor $\mathbf{K} = \mathbf{K}(\mathbf{x}) \in \mathbb{R}^{d\times d}$ are given. For the fracture manifold we search for corresponding reduced quantities S_Γ, P_Γ, and $\mathbf{v}_\Gamma$ satisfying

$$\begin{aligned} &(\omega S_\Gamma)_t + \operatorname{div}_\Gamma\big(\omega \mathbf{v}_\Gamma f^\Gamma(S_\Gamma)\big) = [\![(\mathbf{v}\cdot\mathbf{n}) f(S)]\!], \\ &\operatorname{div}_\Gamma(\omega \mathbf{v}_\Gamma) = [\![\mathbf{v}\cdot\mathbf{n}]\!], \qquad \text{in } \Gamma(t), t>0. \\ &\omega \mathbf{v}_\Gamma = -\lambda^\Gamma(S_\Gamma)\mathbf{K}^\Gamma \nabla_\Gamma(\omega P_\Gamma) \end{aligned} \qquad (11)$$

In (11), we denote given quantities as in (10) but indicate them by a superscript. The brackets $[\![\cdot]\!]$ denote jumps oriented with respect to some specified normal $\mathbf{n}$ of the fracture. To ensure mass conservation and pressure equilibrium across the fracture boundaries appropriate coupling conditions have to be imposed.
Following earlier work on discrete-fracture models we propose in the first part of the talk a formal derivation of the mixed-dimensional model by vertical avaraging from the model with fully-dimensionally fracture geometry. Whereas a rigorous dimension reduction for two-phase flow appears to be difficult we can provide new rigorous results for simplified situations: this includes regularized two-phase systems as well as single hyperbolic equations in cylindric fractures. It turns out that the differential-equation model (11) on Γ is only valid in the limit case where the permeability ratio turns the fracture from a

fluid barrier to a fluid conduct.
As the second new contribution we propose a mass-conserving finite-volume based scheme for the model (10)-(11). Using a recently developped moving-mesh ansatz we can generalize the setting to evolving fractures. Finally we present numerical validations of the formal averaging process and consider some infiltration flows in porous media with bifurcating fractures.
This is joint work with Maximilian Hörl and Taras Mel'nyk.

Increasingly high-order ENO schemes with multi-resolution

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When we solve hyperbolic conservation laws, the difficulty lies in twofold: discontinuities may emergy in finite time even when the initial condition is sufficiently smooth, plus discontinuous solutions are usually accompanied by sophisticated structures with multi-scales. Therefore, high-order numerical schemes with excellent shock-capturing and multi-resolution properties are preferred for solving such problems. The essentially non-oscillatory (ENO) schemes and weighted ENO (WENO) schemes are cutting-edge high-order shock-capturing schemes and achieve great success in practice. In this talk, we will present an efficient class of very high-order (up to 17th-order) essentially non-oscillatory schemes with multi-resolution (ENO-MR) for solving hyperbolic conservation laws. The candidate stencils for constructing ENO-MR schemes range from first-order one-point stencil increasingly up to the designed very high-order stencil. The proposed ENO-MR schemes adopt a very simple and efficient strategy that only requires the computation of the highest-order derivatives of a part of candidate stencils. Besides simplicity and high efficiency, ENO-MR schemes are completely parameter-free and essentially scale-invariant. Theoretical analysis and numerical computations show that ENO-MR schemes achieve designed high-order convergence in smooth regions which may contain high-order critical points (local extrema) and retain ENO property for strong shocks. In addition, ENO-MR schemes could capture complex flow structures very well.

A posteriori error estimates for conservation laws based on the theory of shifts

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We present recent results on a posteriori error estimates for modified Runge-Kutta Discontinuous Galerkin approximations of scalar hyperbolic conservation laws in one space dimension. They are intended as a step towards error control of numerical schemes for systems. Indeed, although we discuss the scalar case, we are careful not to appeal to the Kruzhkov theory for conservation laws.

Instead, our estimates are based on novel quantitative stability estimates that extend the theory of shifts, and in particular, the framework for proving stability first developed by Krupa and Vasseur. The theory of shifts and a-contraction are techniques which adapt well to systems and as a consequence our methods have no inherent small-data limitations and are expected to be extensible to systems. This is the first time this theory has been used for quantitative estimates.

In order to obtain computable a posteriori error estimates from our stability estimates we use suitable reconstructions of numerical solutions.

While we prove reliability of our error estimator, i.e. that it is an upper bound for the error, its efficiency can only be investigated experimentally and we present numerical experiments that show that it scales well under mesh refinement.

The talk covers joint work with Sam G. Krupa (Leipzig).

A moment-based Hermite WENO scheme with unified stencils for hyperbolic conservation laws

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In this talk, we will introduce a fifth-order moment-based Hermite weighted essentially non-oscillatory scheme with unified stencils (termed as HWENO-U) for hyperbolic conservation laws. The main idea of the HWENO-U scheme is to modify the first-order moment by a HWENO limiter only in the time discretizations using the same information of spatial reconstructions, in which the limiter not only overcomes spurious oscillations well, but also ensures the stability of the fully-discrete scheme proved by the von-Neumann analysis. Benefited by this new framework, the HWENO-U scheme involves only a single HWENO reconstruction throughout the entire spatial discretizations, while previous HWENO schemes have to bring additional procedures. Meanwhile, the HWENO-U scheme can use the artificial linear positive weights (the sum is one), but a normalization is made for the original definition of non-linear wights to achieve scale-invariance, which can reduce problem-specific dependencies especially for simulating the problems with sharp scale variations. Compared with previous HWENO schemes, the HWENO-U scheme is simpler and more efficient for utilizing the same candidate stencils, reconstructed polynomials, and nonlinear weights both in the limiter and the spatial reconstruction. Besides, the HWENO-U scheme has more compact stencils, higher resolutions near discontinuities, and smaller numerical errors in smooth regions than a fifth-order WENO scheme with the same framework. Extensive numerical tests are carried out to validate the efficiency, robustness, accuracy, and resolution of the proposed scheme.

Some remarks on concavity of entropy and symmetrization of the full Euler system

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Positive symmetric systems are of interest since there is a local in time existence theory of solutions for such quasilinear systems for sufficiently smooth initial data, see Kato [7] and Hughes et al. [6]. Friedrichs [1] originally introduced this concept for linear systems.

It was first noted by Godunov [2] in a short note and a following paper [3] that the Euler equations of gas dynamics can be brought into such a form via generating potentials for the conserved variables and fluxes, see also Godunov and Romenskii [4] for extensions of the concept. This structure is also central to theoretical frameworks such as extended thermodynamics, see Müller and Ruggeri [9].

An important element is the positivity of the system which relies on the physical entropy H having a negative definite Hessian in the conserved variables of the Euler equations. In two space dimensions this was shown by Harten [5]. For the three dimensional case Lukáčová-Medvid'ová and Yuan [8] gave a proof of lower bounds of the eigenvalues of the Hessian of $-H$ via the characteristic polynomial. The proof is very technical and partially relies on computer algebra. There are various outlines of proofs for the strict convexity of $-H$ in the literature.

But strict convexity is only a necessary but not sufficient for a positive definite Hessian of $-H$, see e.g. [10]. The talk surveys some approaches to proving that the physical entropy density function for the Euler equations in conservative variables in three space dimensions is strictly concave.

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Stiffened gas approximation and GRP resolution for compressible fluid flows of real materials

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The equation of state (EOS) embodies thermodynamic properties of compressible fluid materials and usually has very complicated forms in real engineering applications. The complexity of EOS in form gives rise to the difficulty in analyzing relevant wave patterns and in designing efficient numerical algorithms. In this talk, the generalized Riemann problem (GRP) solver for general EOS is developed by illustrating how the thermodynamical variation is integrated into the design of high resolution methods. The importance of thermodynamic effect is demonstrated numerically in the resolution of strong waves. For the sake of reducing computational costs, a strategy of local stiffened gas approximation is proposed for computing compressible fluid flows of real materials [1]. The stiffened gas EOS is used to approximate general EOSs locally with certain thermodynamic compatibility at each interface of computational control volumes so that the GRP solver can be significantly simplified and the computational cost of the resulting scheme is reduced up to two orders of magnitude. The resulting scheme is demonstrated to be efficient and robust through typical numerical examples which display the excellent performance of such an approximation under extreme thermodynamics.

[1] Wang et al., *J. Sci. Comput.* **95 (1)** (2023).

Estimatable variation neural networks for approximating scalar hyperbolic conservation laws

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The recent successes of neural network based algorithms in a variety of applications has lead to a growing interest in using neural networks to describe physical phenomena. This motivates enforcing neural networks to solve a given system of partial differential equations. To achieve this goal, at least for a class of partial differential equations describing transport phenomena, we introduce a neural network architecture which allows us to approximate the integrals of the network up to arbitrary accuracy. Using networks with this architecture, we are able to define loss functionals for which we can show that minimization leads to convergence to weak admissible solutions of scalar hyperbolic conservation laws in one dimension. Further we prove the existence of networks with arbitrarily small loss under natural assumptions. We present numerical evidence that the minimization of these loss functionals is feasible with standard optimization techniques used in deep learning.

This research was financed by the Mainz Institute for Multiscale Modeling (M^3ODEL) as well as by TRR 165 Waves to Weather. This is a joint work with Mária Lukáčová - Medviďová.

GPGPU-based heterogeneous parallel implementation of direct discontinuous Galerkin methods on the component-based HODG framework

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This presentation provides the CUDA and hybrid CUDA/MPI based on GPGPU heterogeneous parallel strategies for the direct discontinuous method (DDG) on 3D unstructured grids. This implementation was conducted on the component-based HODG framework. Firstly,we present the full single-GPU implementation of the three-dimensional (3D) DDG method with cell-level parallelism. Herein, all the numerical operators including volume integration, face integration(numerical fluxes), solution approximation, and time iteration, are implemented by designing the corresponding kernel functions. Especially,we implement several key memory access optimization strategies, which are crucial for performance improvement.Operators merging and shared memory utilizing reduces the number of global access. Thus, accesses merging and data structure reconstruction enhances the efficiency of global memory access. To align with data access pattern,we employ atomic operations to eliminate data race conditions. Furthermore, we propose a full hybrid GPU/CPU heterogeneous parallel strategy to implement multi-GPU parallelization of the DDG method, where asynchronization optimization is introduced to fully overlap communication and computation and basically eliminates the communication overhead. Finally, several numerical tests are conducted on Tesla V100 Cards to show performance of the parallelization. We utilized the NVIDIA performance testing tool, nvprof, to evaluate multiple metrics of the kernel functions and conducted a detailed analysis of the results. In the tests of parallel scalability, the strong scaling efficiency achieves 97% from 4 to 32 GPU cards, and the weak scaling efficiency is 86% from 4 to 32 GPU cards.

Weak PINNs for Hyperbolic Conservation Laws

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Physics-Informed Neural Networks (PINNs) [1] are a new machine learning method for approximating solutions to partial differential equations. They are based on minimizing a Monte-Carlo approximation of the L^2 norm of the PDE residual. Due to this, they are considered meshfree methods which are suitable for high-dimensional problems. However, for systems of nonlinear hyperbolic conservation laws, we show that standard PINNs fail at approximating discontinuous solutions using explicit computations.

Our computations imply that it is necessary to use weak (dual) norms of the PDE residual instead, in analogy to moving from the strong form of the PDE to the weak formulation for discontinuous solutions. This approach has been termed "weak PINNs" (wPINNs) [2], wherein an adversarial neural network estimates the dual norm.

Extending wPINNs from [2] we approximate the weak norm computation by solving a family of dual elliptic problems with the PDE residual as right-hand side. This makes learning more stable and effective. Similarly, we introduce and compute an entropy residual, which mimics imposing an entropy inequality on the learned solution. Our modified wPINNs also extend naturally to systems of conservation laws.

For the case of scalar nonlinear hyperbolic conservation laws we also outline how to treat boundary conditions in a weak sense using similar techniques and show the efficacy of this strategy. We verify our technique and extensions in numerical experiments for Burgers equation and the compressible Euler equations, both in one space dimension.

Lastly, we investigate the performance of wPINNs for higher-dimensional problems. We show numerical evidence that even in two dimensions, outside of some basic examples, the performance of wPINNs is significantly worse than in one dimension. We believe this is due to difficulties with quadrature formulas that are used when computing the dual norm. These difficulties stem from the residual being concentrated close to discontinuities in the solution. This may imply that the PINNs approach, despite being meshfree, is not suitable for high-dimensional hyperbolic conservation laws when discontinuities are present.

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Parallel Session F2 (SF)

PhD Student Forum
East Middle Hall-Building 1-202,
14:00-17:40

Long-time asymptotic equivalence between the Vlasov-Poisson-Boltzmann equations and the isentropic compressible Navier-Stokes-Poisson equations

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In this talk, we are concerned with the long-time asymptotic equivalence between the global classical solution of the Vlasov-Poisson-Boltzmann (VPB) system and that of the isentropic Navier-Stokes-Poisson (NSP) system. Li-Yang-Zhong (Arch. Ration. Mech. Anal. 241 (2021), no. 1, 311-355.) and Li-Matsumura-Zhang (Arch. Ration. Mech. Anal. 196 (2010), no. 2, 681-713.) demonstrated that the global solutions of the VPB equations and the NSP equations exhibit optimal decay rates of $(1+t)^{-\frac{1}{4}}$, respectively.

By demonstrating the mutual cancellation of the slowest decaying components in the two equations, the global solution of the VPB system with small initial data are proved to be asymptotically equivalent (mod decay rate $(1+t)^{-\frac{3}{4}}\ln(1+t)$) to that of the NSP equations for the corresponding initial data. Two coefficients a and b in the NSP system are introduced to make the eigenvalues of the NSP equations consistent with those of the VPB equations up to $O(|\xi|^2)$ at low frequency. As an application, we show that the momentum of the global solution for the linearized Vlasov-Maxwell-Boltzmann system exhibits asymptotically equivalence (mod decay rate $(1+t)^{-\frac{3}{4}}$) to that of the linearized Navier-Stokes-Maxwell equations.

A First-order Hyperbolic Approximation of the Navier-Stokes-Cahn-Hilliard System for Incompressible Two-phase Flows

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In this project we consider the dynamics of two incompressible and immiscible fluids in the convection-dominated regime at constant temperature. For the interfacial dynamics we employ a diffuse-interface approach. The Navier-Stokes-Cahn-Hilliard (NSCH) system is widely used to model the evolution of interfaces in two-phase flows. However, the fourth-order differential operator in the NSCH system results in high computational costs. Therefore, we focus on the NSCH system in a different way and suggest a low-order approximation, which is obtained in the following way. First, artificial compressibility is used in the incompressibility constraint, and the Euler-part of the NS equations is converted to a first-order system. Then, the CH equation is approximated by the Euler-Korteweg (EK) system with friction. It is proved in that solutions of the EK system with friction converges to solutions of the CH equation in the limit of infinite friction. Further, the NS system and the EK system with friction are coupled in a thermodynamically consistent way i.e., such that the total energy of the coupled system decreases in time. The aim of the coupling is to obtain a first-order hyperbolic system of equations. Up to our knowledge, coupling of the NS-EK system with friction is a new approach for the diffuse-interface ansatz. Furthermore, we need to apply a relaxation approach so that we can reduce third-order derivatives in the coupled system to first-order derivatives. Thus, we obtain a first-order system, (neglecting the viscosity term). To analyse the hyperbolicity of the new system, the eigenvalues of the Jacobian of flux are computed. The characteristic analysis of the system shows that it is strictly hyperbolic. Having a hyperbolic system of equations allows us to use tailored numerical methods. In order to solve the new system we use a high-order discontinuous Galerkin method. Finally, the numerical experiments for merging droplets are performed with the first-order hyperbolic system and the original NSCH system and the results are compared.

Keywords: Navier-Stokes-Cahn-Hilliard system, Euler-Korteweg system with friction, first-order hyperbolic system, diffuse-interface approach, two-phase flows

Convergence of the relaxed compressible Navier-Stokes equations to the incompressible Navier-Stokes equations

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In this talk, I will present the combined low Mach number and the relaxation limits for the isentropic compressible Navier-Stokes equations for which the classical Newtonian law of viscosity is replaced by a revised Maxwell's law satisfying Galilean invariance. It is shown that the solutions of the relaxed compressible Navier-Stokes equations converge to that of the incompressible Navier-Stokes equations as the Mach number and relaxation parameters tend to zero. This talk is based on joint work with Prof. Yuxi Hu and Prof. Qiangchang Ju.

Initial-boundary value problem for 2D temperature-dependent tropical climate model

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It is well known that the tropical climate model is an important model to describe the interaction of large scale flow fields and precipitation in the tropical atmosphere. In this paper, we address the issue of global well-posedness for 2D temperature-dependent tropical climate model in a smooth bounded domain. Through classical energy estimates and De Giorgi-Nash-Moser iteration method, we obtain the global existence and uniqueness of strong solution in classical energy spaces. Compared with Cauchy problem, we establish more delicate a priori estimates with exponential decay rates. To the best of our knowledge, this is the first result concerning the global well-posedness for the initial-boundary value problem in 2D tropical climate model.

On the incompressible and non-resistive limit of 3D compressible magnetohydrodynamic equations in bounded domains

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In this paper, we investigate the incompressible and non-resistive limit for the initial boundary value problem of isentropic compressible resistive magnetohydrodynamic equations with ill-prepared initial data in three-dimensional bounded domains. We establish the higher-order uniform estimates with respect to both the Mach number and the resistivity coefficient in the framework of new type of weighted Sobolev spaces. Then we obtain the strong convergence of the magnetic field and the divergence-free component of the velocity field, as both the Mach number and the resistivity coefficient tend to zero.

On Uniqueness of Steady 1-D Shock Solutions in a Finite Nozzle via Asymptotic Analysis for Physical Parameters

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In this paper, we study the uniqueness of the steady 1-D shock solutions for the inviscid compressible Euler system in a finite nozzle via asymptotic analysis for physical parameters. The parameters for the heat conductivity and the temperature-depending viscosity are investigated for both barotropic gases and polytropic gases. It finally turns out that the hypotheses on the physical effects have significant influences on the asymptotic behaviors as the parameters vanish. In particular, the positions of the shock front for the limit shock solution(if exists) are different for different hypotheses. Hence, it seems impossible to figure out a criterion selecting the unique shock solution within the framework of the inviscid Euler flows.

Thermal and Momentum Boundary Layers in the Hydromagnetic Hybrid Nanofluid Flow in a Conical Gap Embedded in a Porous Medium

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This study examines heat transfer in a plate-cone viscometer using a semi-analytical approach to analyze the impact of porous media on nanofluid flow and heat transfer within a co-rotating conical gap between a disk and a cone. Factors such as Cattaneo–Christov heat flux, radiative heat, viscous dissipation, and Joule heating are considered, with the fluid being an electrically conducting hybrid nanofluid under a transverse magnetic field. Non-linear partial differential equations are transformed into ordinary differential equations using similarity variables and solved using the Homotopy analysis method, with results presented through graphs and tables. Findings indicate that increasing porosity and magnetic field significantly enhance thermal transmission rates without altering surface temperature in disk-cone configurations.

Global solutions to the Cauchy problem of viscous shallow water equations in some classes of large data

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We consider the Cauchy problem of shallow water equations under partial smallness assumption of the initial data and obtain the global existence and decaying estimates. We apply Green's function method, together with the regularity criterion and the abstract bootstrap argument which have been widely used in the previous work. Our paper shows that based on the precise information about the different frequency parts in Green's function and the resulted time decaying structures, one could give a concise and clear proof of the global existence of the solution and also obtain its decaying estimates..

Parallel Session G2 (SF)

PhD Student Forum
East Middle Hall-Building 1-301,
14:00-18:05

Homogeneous Dirichlet problem for degenerate parabolic-hyperbolic PDE driven by Lévy noise

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In this article, we study the homogeneous Dirichlet problem for a degenerate parabolic-hyperbolic PDE perturbed by Lévy noise. We introduce the definition of entropy solution and prove the existence and uniqueness of entropy solution. As usual, the existence is obtained via classical viscosity arguments. Due to the degeneracy in the equation, one needs to get the point-wise convergence of the sequence of viscous solutions. To do so, we first prove the uniqueness of Young measure-valued limit processes of viscous solutions using an appropriate version of the Kružkov's semi-entropy formulation. In fact, the main arguments are based on obtaining the local and global Kato's inequalities, allowing comparison principle in L^1. In comparison to existing literature concerning the well-posedness of entropy solutions for the Dirichlet problem for conservation laws driven by Brownian noise, our present analysis involves a simpler approach to obtain the global Kato's inequality. The work is jointly conducted by Soumya Ranjan Behera and Dr. Ananta K. Majee; Department of Mathematics, IIT, Delhi.

Non-uniqueness of the forced stochastic Navier-Stokes equations

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In this talk, we consider the 3D forced stochastic Navier-Stokes equation driven by additive noise. By constructing an appropriate forcing term, we prove that there exist distinct Leray solutions in the probabilistically weak sense, which implies the joint uniqueness in law fails in the Leray class. Furthermore, we show that the Lions exponent is the sharp viscosity threshold for the uniqueness/non-uniqueness in law of Leray solutions.

Large Deviation Principle for pseudo-monotone evolutionary equation

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We establish Freidlin-Wentzell's large deviation principle for a stochastic partial differential equation with the nonlinear diffusion-convection operator in divergence form satisfying p-type growth, involving coercivity assumptions, perturbed by small multiplicative Brownian noise. We utilize the weak convergence method to prove the Laplace principle, which is equivalent to the large deviation principle in our framework. The main technical difficulties arise from the p-type growth of the operator associated with Lipschitz continuous perturbation. The well-posedness of an associated deterministic problem is established by using time discretization and Minty-type monotonicity argument. We use Girsanov's theorem and Skorokhod representation theorem to handle the stochastic counterpart.

The global existence of martingale solutions to stochastic compressible Navier-Stokes equations with density-dependent viscosity

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In this talk, we study the global existence of martingale solutions to the compressible Navier-Stokes equations driven by stochastic external forces, with density-dependent viscosity and vacuum. This work can be regarded as a stochastic version of the deterministic Navier-Stokes equations (Vasseur-Yu, Invent. Math., 206:935–974, 2016.), in which the global existence of weak solutions was established for adiabatic exponent $\gamma > 1$. For the stochastic case, the regularity of density and velocity is even worse for passing the limit in nonlinear terms. We design a regularized system to approximate the original system. To make up for the lack of regularity of velocity, we need to add an artificial Rayleigh damping term besides the artificial viscosity and damping forces in (Vasseur-Yu, Invent. Math., 206:935–974, 2016.). Moreover, we have to send the artificial terms to 0 in a different order. This is a joint work with Prof. Yachun Li.

Entropy conserving/stable schemes for vector-kinetic and macroscopic models

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In the realm of numerical methods for hyperbolic partial differential equations (PDEs), achieving entropy conservation or stability is a crucial yet challenging endeavour. While existing literature predominantly focuses on entropy conserving/stable schemes for macroscopic models, such schemes do not ensure the entropy conservation/stability for vector-kinetic models. In this work, we address this gap by proposing an entropy conserving scheme tailored specifically for vector-kinetic models. We demonstrate that the moment of this scheme results in an entropy conserving scheme for the corresponding macroscopic model. Drawing inspiration from established methods in the field, we develop an entropy stable scheme for the vector-kinetic model, with the numerical viscosity of the entropy conserving scheme serving as a crucial reference point, akin to the seminal work by [1]. We also demonstrate that the moment of this scheme results in an entropy stable scheme for the corresponding macroscopic model. To validate the efficacy of our approach, we conduct extensive numerical experiments on benchmark test problems, including scalar and shallow water equations. Our results confirm the conservation and stability of both vector-kinetic and macroscopic entropies, providing comprehensive insights into the performance and reliability of the developed schemes.

This work has been recently published in a journal (see [2]).

[1] Eitan Tadmor, *The Numerical Viscosity of Entropy Stable Schemes for Systems of Conservation Laws. I.* , Mathematics of Computation, *49 (179)*, 91–103, 1987.

[2] Megala Anandan and S. V. Raghurama Rao, *Entropy conserving/stable schemes for a vector-kinetic model of hyperbolic systems*, Applied Mathematics and Computation, *465 (128410)*, 15 March 2024.

Well posedness and limit theorems for a class of stochastic dyadic models

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We consider stochastic inviscid dyadic models with energy-preserving noise. It is shown that the models admit weak solutions which are unique in law. Under a certain scaling limit of the noise, the stochastic models converge weakly to a deterministic viscous dyadic model, for which we provide explicit convergence rates in terms of the parameters of noise. A central limit theorem underlying such scaling limit is also established. In case that the stochastic dyadic model is viscous, we show the phenomenon of dissipation enhancement for suitably chosen noise.

Global pathwise solution in probability for an abstract stochastic equations

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We establish the existence and uniqueness of maximal pathwise solution of an abstract nonlinear stochastic equations forced by a multiplicative white noise, and show the above pathwise solution is global in a positive probability provided the initial data is sufficiently small. Then applying these results to the initial boundary value problem of the stochastic Navier-Stokes equations. Furthermore, we consider the global pathwise solution which evolving in H^m for the stochastic Navier-Stokes equations on torus.

Nonlinear stability of planar traveling waves to a multidimensional hyperbolic-parabolic system arising from chemotaxis

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In this talk we present our recent study on the nonlinear stability of planar traveling waves to a three-dimensional hyperbolic-parabolic system on $\mathbb{R}\times\mathbb{T}^2$ under general perturbations. The system comes from a chemotaxis model with a logarithmic sensitivity that describes the initiation of tumor angiogenesis. We show that the planar traveling waves with small amplitude are nonlinearly stable under general initial perturbations. The proof is based on a combination of the a-contraction technique with a time-dependent shift first introduced by Kang and Vasseur for the Navier-Stokes system, and some essential cancellation structures of the chemotaxis model.

Well-posedness and regularization-by-noise for the stochastic Prandtl equation

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In this talk, I will present our recent results on the well-posedness and regularization-by-noise phenomenon for the stochastic Prandtl equation which governs the velocity field inside the boundary layer that appears in the inviscid limit of the stochastic Navier-Stokes equation. Analogous to the deterministic case, the stochastic Prandtl equation is physically derived via the method of multi-scale analysis. New problem arises when establishing well-posedness in the stochastic situation: one can never derive a pathwise control of the energy functional which is used to describe the analytic radius of the solution in the deterministic setting. To this end, we establish higher-order estimates in conormal Sobolev spaces in order to get rid of the dependence of the analytic radius on the unknown. We choose approximate scheme which fits into the stochastic framework and the well-posedness is finally constructed in a tangentially analytic and normally Sobolev-type space. We also study the regularization-by-noise phenomenon for the stochastic Prandtl equation driven by a tangential random diffusion. It turns out that the noise regularizes the equation in the sense that there exists a global Gevrey-2 solution with high probability and the radius growing linearly in time.

7 July 4 Thursday

Parallel Session B4 (CT)

Contributed Talks
East Middle Hall-Building 1-109,
11:10-18:05

Structure stability and asymptotic limits of the subsonic solutions to sonic boundary problem of the steady Euler-Poisson system

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For the stationary hydrodynamic model for semiconductors with sonic boundary, represented by Euler-Poisson equations, it possesses the various physical solutions including interior subsonic solutions/interior supersonic solutions/shock transonic solutions/C^1-smooth transonic solutions. However, the structural stability and asymptotic limits for these physical solutions are challenging and have remained open as we know. In this talk, we give the structural stability and the asymptotic limits of interior subsonic solutions when the doping profiles are restricted in the subsonic region. The main result is proved by using the local (weighted) singularity analysis and the monotonicity argument. Both the result itself and techniques developed here will give us some truly enlightening insights into our follow-up study on the structural stability of the remaining types of solutions. This is a joint work with Haifeng Hu, Ming Mei, Yuejun Peng and Guojing Zhang.

Asymptotic stability of stationary solutions to outflow problem for compressible viscoelastic system in one dimensional half space

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We consider the compressible viscoelastic system in a one dimensional half space $\mathbb{R}_+ = (0,\infty)$:

$$\rho_t + (\rho v)_x = 0, \tag{12}$$

$$(\rho v)_t + (\rho v^2)_x - \nu v_{xx} + (P(\rho))_x = \beta^2(\rho F^2)_x, \tag{13}$$

$$F_t + vF_x = v_x F. \tag{14}$$

with the initial condition and boundary conditions at $x = \infty$ and $x = 0$:

$$(\rho, v, F)|_{t=0} = (\rho_0, v_0, F_0), \ \inf_{x\in\mathbb{R}_+} \rho_0(x) > 0, \ \rho_0 F_0 = \rho_+ F_+ \tag{15}$$

$$\lim_{x\to\infty} (\rho, v, F) = (\rho_+, v_+, F_+), \ v(t,0) = v_b. \tag{16}$$

Here $\rho = \rho(t,x)$, $v = v(t,x)$, and $F = F(t,x)$ are the unknown density, velocity field, and deformation tensor, respectively, at time $t \geq 0$ and position $x \in \mathbb{R}_+$; $P(\rho)$ stands for the pressure assumed to be a smooth function of ρ satisfying $P'(\rho) > 0$ and $P''(\rho) > 0$ for $\rho > 0$; $\nu > 0$ is the viscosity coefficient; $\beta > 0$ is the strength of the elasticity; the end states ρ_+, v_+ and F_+ are given constants with $\rho_+ > 0$, and v_b is a given constant assumed to be $v_b < 0$ for studying the situation that the fluid runs out from the boundary $\{x = 0\}$.

We investigate the existence and stability of the stationary solutions of the problem (12)–(16) denoted by $(\tilde\rho(x), \tilde v(x), \tilde F(x))$. Let $M_+ := |v_+|/c_+$ be the Mach number, where $c_+ := \sqrt{P'(\rho_+)}$ stands for sound speed. We set $\beta_c := c_+\sqrt{M_+^2 - 1}/|F_+|$ for the case $M_+ > 1$. The existence of stationary solutions is established as the following proposition.

Proposition. *There exists a negative constant $v_* < 0$ such that the problem (12)–(16) has a stationary solution $(\tilde\rho(x), \tilde v(x), \tilde F(x))$ if and only if $M_+ > 1$, $0 < \beta \leq \beta_c$ and $v_b < v_*$ hold. In addition the obtained stationary solution satisfies*

$$|(\tilde\rho(x), \tilde v(x), \tilde F(x)) - (\rho_+, v_+, F_+)| \leq C \begin{cases} \delta e^{-cx}, & 0 \leq \beta < \beta_c, \\ \delta(1+\delta x)^{-1}, & \beta = \beta_c, \end{cases}$$

where δ denotes $\delta := |v_b - v_+|$.

Concerning to the stability of the stationary solution, we have the following theorem.

Theorem. *Suppose that at least one of the following two cases holds:*

(ND) $0 < \beta < \beta_c$ *and* $v_b < v_*$,

(D) $\beta = \beta_c$, $1 < M_+ < \sqrt{\dfrac{\rho_+ P''(\rho_+)}{2P'(\rho_+)} + 1}$ *and* $v_b < v_+$.

Then, there exists a positive small number $\varepsilon_0 > 0$ such that if (ρ_0, v_0, F_0) and δ satisfy

$$\|(\rho_0 - \tilde\rho, v_0 - \tilde v, F_0 - \tilde F)\|_{H^2} + \delta \leq \varepsilon_0,$$

then the stationary solution is asymptotically stable.

This study is based on a joint work with Yoshihiro Ueda (Kobe University).

Eckhaus instability of compressible Taylor vortices

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This talk is concerned with the bifurcation and stability of the compressible Taylor vortex. Consider the compressible Navier-Stokes equations in a domain between two concentric infinite cylinders. If the outer cylinder is at rest and the inner one rotates with sufficiently small angular velocity, a laminar flow, called the Couette flow, is stable. When the angular velocity of the inner cylinder increases, beyond a certain value of the angular velocity, the Couette flow becomes unstable and a vortex pattern, called the Taylor vortex, bifurcates and is observed stably. This phenomenon is mathematically formulated as a bifurcation and stability problem. In this talk, the compressible Taylor vortices are shown to bifurcate near the criticality for the incompressible problem when the Mach number is sufficiently small. The localized stability of the compressible Taylor vortices is considered under axisymmetric perturbations and it is shown that the Eckhaus instability of compressible Taylor vortices occurs as in the case of the incompressible ones.

Large asymptotic behavior of solutions to bipolar Euler-Poisson equations with time-dependent damping

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In this talk, we consider the Cauchy problem for the 1D Euler / Euler-Poisson equations with time-dependent damping. The damping coefficient is $-\frac{\mu}{(1+t)^\lambda}$.

Firstly, we consider the large time behaviors of solutions to the Euler / Euler-Poisson equations with time-dependent damping for $\mu = 1, -1 < \lambda < 1$. We show that the system has a couple of global solutions uniquely, and such solutions tend time-asymptotically to the shifted nonlinear diffusion waves, which are the solutions of the corresponding nonlinear parabolic equation governed by the Darcy's law. We further derive the optimal convergence rates when the initial perturbations are in L^2.

Secondly, we investigate the large time behaviors of solutions to the Euler / Euler-Poisson equations with critical overdamping for $\mu = 1, \lambda = -1$. We show that the system still admits nonlinear diffusion phenomenon, and derive the optimal convergence rate in logarithmic form.

Finally, we consider the global and blow-up solutions to the Euler equations for $\lambda > 0$ in C space. For $0 < \lambda < 1$ and $\mu > 0$, or $\lambda = 1$ but $\mu > 2$, the solutions are proved to exist globally in time, when the derivatives of the initial data are small, but the initial data themselves can be arbitrarily large. While, when the initial Riemann invariants are monotonic and their derivatives with absolute value are large at least at one point, then the solutions are still bounded, but their derivatives will blow up at finite time. For $\lambda > 1$ and $\mu > 0$, or $\lambda = 1$ but $0 < \mu \leq 1$, the derivatives of solutions will blow up for all initial data.

Stability of Stationary Solutions to the Non-isentropic Euler–Poisson System in a Perturbed Half Space

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The main concern of the talk is to mathematically investigate the formation of a plasma sheath near the surface of nonplanar walls. We study the existence and asymptotic stability of stationary solutions for the nonisentropic Euler-Poisson equations in a domain of which boundary is drawn by a graph, by employing a space weighted energy method. Moreover, the convergence rate of the solution toward the stationary solution is obtained, provided that the initial perturbation belongs to the weighted Sobolev space. Because the domain is the perturbed half space, we first show the time-global solvability of the nonisentropic Euler-Poisson equations, then construct stationary solutions by using the time-global solutions. This is a joint work with Professor Masahiro Suzuki.

Asymptotic stability for n-dimensional magnetohydrodynamic equations

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This talk is concerned with the stability theory of n-dimensional incompressible and compressible magnetohydrodynamic (MHD for short) equations with only kinematic viscosity or magnetic diffusion in the periodic domain $\mathbb{T}^n$. I will present some new results on the asymptotic stability and sharp decay estimates of this system when the magnetic field close to an equilibrium state satisfying the Diophantine condition. In the present works, by exploiting and effectively utilizing the structure of perturbation system, a new dissipative mechanism is found out and applied so that we can sharply improve the spaces of existing works, where the decay estimates and asymptotic stability of solutions are taking place . Some key ideas of our method will be discussed. This talk is based on joint works with Quansen Jiu and Yaowei Xie.

Some progress on the Tropical Climate Model without thermal diffusion

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In this talk, we are concerned with a series of results including the well-posedness, stability and optimal large-time behavior of the 2D or 3D Tropical climate model without thermal diffusion. Compared with the previous results, we exploit the structure of the thermal function and the first baroclinic mode of the velocity fields to overcome the difficutly arised from the lack of the thermal dissipation.

Asymptotic behavior of compressible non-isothermal nematic liquid crystal flow in infinite layer

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In this talk, we consider the flow of compressible nematic liquid crystal in an infinite layer, which is governed by the following system based on simplified Ericksen-Leslie system:

$$\partial_t \rho + \mathrm{div}\,(\rho \boldsymbol{u}) = 0, \quad (17)$$

$$\partial_t(\rho \boldsymbol{u}) + \mathrm{div}\,(\rho \boldsymbol{u} \otimes \boldsymbol{u}) = \mathrm{div}\,\mathbb{S}_C, \quad (18)$$

$$\partial_t(\rho\theta) + \mathrm{div}\,(\rho\theta \boldsymbol{u}) + \mathrm{div}\,\boldsymbol{q} = \mathbb{S}_C : \nabla \boldsymbol{u}, \quad (19)$$

$$\partial_t \boldsymbol{d} + \boldsymbol{u}\cdot\nabla \boldsymbol{d} = \tau^*(\Delta \boldsymbol{d} + |\nabla \boldsymbol{d}|^2 \boldsymbol{d}). \quad (20)$$

Here the unknown variables ρ, $\boldsymbol{u}$, θ denote the density, velocity and temperature of fluid, respectively, $\boldsymbol{d}$ the orientation field for averaged macroscopic molecular directions of nematic liquid crystal, and hence, $|\boldsymbol{d}| = 1$. $\boldsymbol{q}$ is the heat flux given by $\boldsymbol{q} = -\kappa^*\nabla\theta$. Moreover, $\mathbb{S}_C = \mathbb{S}_N - \eta^*(\nabla \boldsymbol{d} \odot \nabla \boldsymbol{d} - \frac{1}{2}|\nabla \boldsymbol{d}|^2\mathbb{I}) - P\mathbb{I}$, where $P = P(\rho,\theta)$ is pressure which satisfies $\partial_\rho P \geq 0$, $\partial_\theta P \geq 0$, and $\mathbb{S}_N = \mu(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^\top) + \mu'(\mathrm{div}\,\boldsymbol{u})\mathbb{I}$. η^*, μ, μ', κ^* and τ^* describe some positive constants. $\mathbb{I}$ is the identity matrix. We set $(\boldsymbol{u}\otimes\boldsymbol{u})_{ij} = u^i u^j$, $(\nabla \boldsymbol{d}\odot\nabla \boldsymbol{d})_{ij} = \partial_{x_i}\boldsymbol{d}\cdot\partial_{x_j}\boldsymbol{d}$ and $\mathbb{S}_C : \nabla \boldsymbol{u} = \sum_{i,j}(\mathbb{S}_C)_{ij}\partial_{x_i}u^j$. (17)–(20) is considered in $\Omega_h = \{x = (x', x_3);\, x' = (x_1, x_2),\, 0 < x_3 < h\}$. The boundary condition is

$$\boldsymbol{u}|_{x_3=0,h} = \boldsymbol{0},\ \theta|_{x_3=0} = \theta_0^*,\ \theta|_{x_3=h} = \theta_1^*, \quad \frac{\partial \boldsymbol{d}}{\partial x_3}|_{x_3=0,h} = \boldsymbol{0}, \quad (21)$$

for given positive constants θ_0^*, θ_1^*. We consider density ρ and temperature θ around ρ^* and θ_0^*, respectively, where ρ^* is given positive constant.

The Ericksen-Leslie system is produced by Ericksen [1,2,3] and Leslie [4,5]. The isothermal simplified model is first proposed by Lin [6] and many works have done for this isothermal model. On the other hand, there are few results for non-isothermal model of nematic liquid crystals for global classical solutions.

We prove the existence of global strong solutions and its decay rates when the initial data is close to steady state. It turns out that the low-frequency part of the solution decays like a 2-dimensional heat kernel. We can also see that the low-frequency part appears in the asymptotic reading part of the solution which is affected by nonlinear terms as t goes to infinity.

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The rigidity of steady solutions of Navier-Stokes system and its applications

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The Liouville type theorem for stationary Navier-Stokes system in the whole space is longstanding open problem. In this talk, we first discuss the rigidity of steady Navier-Stokes system with dimension bigger than three in a class more general than self-similar solutions, where we do not need any type of self-similarity or smallness of solutions. Furthermore, this rigidity result is used to study the regularity and far field behavior of steady solutions of high dimensional Navier-Stokes system. Finally, we discuss the rigidity for steady Navier-Stokes system in domains with physical boundaries.

Time-periodic Solutions of the Compressible Euler Equations

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We prove the existence of “pure tone” nonlinear sound waves of all frequencies. These are smooth, space and time periodic, oscillatory solutions of the 3×3 compressible Euler equations in one space dimension. Being perturbations of solutions of a linear wave equation, they provide a rigorous justification for the centuries old theory of Acoustics. In particular, Riemann’s celebrated 1860 proof that compressions always form shocks holds for isentropic and barotropic flows, but for generic entropy profiles, shock-free periodic solutions containing nontrivial compressions and rarefactions exist for every wavenumber k.

Joint work with Blake Temple.

Nonlinear asymptotic stability of vortex sheets with viscosity

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In this talk, we can see that although a vortex sheet is not an asymptotic attractor for the compressible Navier-Stokes equations, a viscous wave that approximates the vortex sheet can be computed explicitly. It is shown that regardless of the strength of vortex sheet, the associated viscous wave is time-asymptotically stable in the L^∞-norm with small initial perturbations for the compressible Navier-Stokes equations.

Parallel Session C4 (CT)

Contributed Talks

East Middle Hall-Building 2-203, 11:10-18:05

A nodal ghost method based on variational formulation and regular square grid for elliptic problems on arbitrary domains and applications to biological network formation

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This talk focuses on the numerical solution of elliptic partial differential equations (PDEs), specifically addressing the challenges arising from irregular domains. Both finite element method (FEM) and finite difference method (FDM), face difficulties in dealing with arbitrary domains. We introduce a novel nodal symmetric ghost method based on a variational formulation approach, which combines the advantages of FEM and FDM. The convergence rates are validated with many numerical experiments, in both one and two space dimensions. At the end, we show an application to biological network formation, computing the solution of a reaction-diffusion equation for the conductivity tensor, coupled with a Poisson equation for the pressure of the fluid.

A PDE-based Bellman Equation for Continuous Time Reinforcement Learning

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In this talk, we address the problem of continuous-time reinforcement learning in scenarios where the dynamics follow a stochastic differential equation. When the underlying dynamics remain unknown and we have access only to discrete-time information, how can we effectively conduct policy evaluation? We first highlight that the commonly used Bellman equation is not always a reliable approximation to the true value function. We then introduce PhiBE, a PDE-based Bellman equation that offers a more accurate approximation to the true value function, especially in scenarios where the underlying dynamics change slowly. Moreover, we extend PhiBE to higher orders, providing increasingly accurate approximations. Additionally, we present a model-free algorithm to solve PhiBE when only discrete-time trajectory data is available. Numerical experiments are provided to validate the theoretical guarantees we propose.

Poisson quadrature method of moments for 2D kinetic equations with velocity of constant magnitude

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This work is concerned with kinetic equations with velocity of constant magnitude. We propose a quadrature method of moments based on the Poisson kernel, called Poisson-EQMOM. The derived moment closure systems are well defined for all physically relevant moments and the resultant approximations of the distribution function converge as the number of moments goes to infinity. The convergence makes our method stand out from most existing moment methods. Moreover, we devise a delicate moment inversion algorithm. As an application, the Vicsek model is studied for overdamped active particles. Then the Poisson-EQMOM is validated with a series of numerical tests including spatially homogeneous, one-dimensional and two-dimensional problems. This is a joint work with Yihong Chen, Ruixi Zhang and Prof. Wen-An Yong at Tsinghua University, China.

Uniform error estimate of a bi-fidelity method for a kinetic-fluid multi-phase flow system with random inputs

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Uniform error estimates of a bi-fidelity method for a kinetic-fluid coupled model with random initial inputs in the fine particle regime are proved in this work. Such a model is a system coupling the incompressible Navier–Stokes equations to the Vlasov–Fokker–Planck equations for a mixture of the flows with *distinct* particle sizes. The main analytic tool is the hypocoercivity analysis for the multi-phase Navier-Stokes-Vlasov-Fokker-Planck system with uncertainties, considering solutions in a perturbative setting near the global equilibrium. This allows us to obtain the error estimates in both kinetic and hydrodynamic regimes. If we choose the low-fidelity model the same kinetic-fluid equation as the high-fidelity model and solve it on a coarser physical mesh than that used for the high-fidelity model under the constraint of the CFL condition, numerical results show the efficiency and accuracy of the bi-fidelity method. This work provides a range of low-fidelity models suitable for the bi-fidelity method, which allows effective handling of challenges posed by high-dimensionality, nonlinearity, coupling and randomness in computation when solving kinetic equations with random parameters. The talk is based on a joint work with S. Jin.

High-order spectral volume gas kinetic scheme

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This report introduces the research progress on high-order compact gas kinetic schemes for compressible fluids, known as Spectral Volume Gas Kinetic Scheme (SVGKS). Firstly, based on the average values of control volumes in the spectral volume element, high-order polynomials are reconstructed in the spectral volume unit, and a new high-precision, high-resolution, and efficient limiter for the spectral volume method is developed using Simple WENO method. Secondly, by combining high-order gas kinetic method, high-order approximations of the velocity distribution functions under continuous and discontinuous conditions are given, respectively. And accordingly, the continuous flux at the internal continuous boundaries of the control volume, and discontinuous numerical fluxes at the spectral volume unit interface and the troubled control volume interface are given. Continuous flux can simplify calculations and save computation time, thus making the SV scheme more competitive. Thirdly, an arbitrary high-order compact gas kinetic method is developed for the quasi-conservative extended Euler equations of compressible multi-component flows with a stiffened gas equations of state(EOS). The high-order compact SV scheme is directly applied to the conservative part. In order to handle the state equation of stiffened gas and avoid numerical oscillations of the gas kinetic scheme at the material interface, the flux of the original gas kinetic scheme is modified. And based on the quasi-conservation method, a high-order compact scheme for the non conservative part is obtained. Numerical tests demonstrate the good performance and the efficiency of the new scheme.

Mathematical modeling and simulation of

mechano-chemical effect on two-phase avascular tumor

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We derive the mathematical model that allows chemotaxis in avascular tumour growth in a two-phase medium. The two phases are the viscous cell phase and the inviscid fluid phase. The conservation of mass-momentum is incorporated in each phase, and appropriate constitutive laws are applied to formulate the governing equations. Further, these equations are simplified into three main variables : cell volume fraction, cell velocity, and nutrient concentration. These variables generate a coupled system of non-linear partial differential equations. A numerical scheme based on the finite volume method is applied to approximate the solution of cell volume fraction. The finite element method is applied to approximate the solutions of cell velocity and nutrient concentration. We investigate tumour growth when when tumour cells move along a fluid containing a diffusible nutrient to which the cells are drawn. We perform some numerical simulations to show the effect of the parameters. The findings of this literature are compatible with the existing literature.

On the dynamical low-rank numerical method for kinetic equations

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The numerical solution of kinetic equations often requires a high computational effort and memory cost due to the potentially six-dimensional phase space. One approach to overcome this difficulty is the reduced order method dynamical low-rank approximation (DLRA). It has recently gained an increasing interest as it has been shown to provide accurate numerical solutions of kinetic PDEs in various applications while reducing the computational time significantly.

This talk will focus on a research project that has the goal to devise an efficient numerical method for solving a BGK-type kinetic equation

$$\partial_t f + v \cdot \nabla_x f = \sigma(M - f).$$

Our approach has the potential to bring about dramatic savings in computational time. We build on the low-rank approximation technique used by Lukas Einkemmer [1]. We show how we have made progress in reducing the numerical effort even further by proving stability estimates for a related system of kinetic equations [2].

This is joint work together with Christian Klingenberg (Wuerzburg, Germany), Lukas Einkemmer (Innsbruck, Austria) and Jonas Kusch (Ås, Norway).

[1] Einkemmer, L., Hu, J., Ying, L.: *An efficient dynamical low-rank algorithm for the Boltzmann-BGK equation close to the compressible viscous flow regime.* In: SIAM J. Sci. Comput., **43**, B1057-B1080. 2021.

[2] Baumann, L., Einkemmer, L., Klingenberg, C., Kusch, J.: *Energy stable and conservative dynamical low-rank approximation for the Su-Olson problem.* In: SIAM J. Sci. Comput., 2024.

Mitigating the ray effect in discrete ordinate method

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The Discrete Ordinates Method (DOM) is the most widely used velocity discretization method for simulating the radiative transport equation. The ray effect stands as a long-standing drawback of DOM. In benchmark tests displaying the ray effect, we observe low regularity in velocity within the solution. To address this issue, we propose a random ordinate method (ROM) to mitigate the ray effect. Compared with other strategies proposed in the literature for mitigating the ray effect, ROM offers several advantages: 1) the computational cost is comparable to DOM; 2) it is simple and requires minimal changes to existing code based on DOM; 3) it is easily parallelizable and independent of the problem setup. Analytical results are presented for the convergence orders of the error and bias, and numerical tests demonstrate its effectiveness in mitigating the ray effect.

Negative particle method for the many-body quantum kinetic equation

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The physical interest in quantum kinetic theory has increased the advances in the study of cold Bose gases in Bose–Einstein condensation and cold Fermi gases, as well as modelling degenerate plasmas during compression of the cold fuel and capsule shell in inertial confinement fusion. However, it is notoriously difficult to solve the quantum kinetic equation numerically due to the well-known curse of dimensionality. In this talk, we would like to discuss a class of negative particle method for solving many-body quantum kinetic equation, based on its branching random walk interpretation associated with negative weights. To alleviate the numerical sign problem inherited in the negative particle method, we suggest two approaches: The first is a variance reduction technique based on the stationary phase approximation to kill the redundant stochastic particles. The second is an adaptive particle annihilation algorithm, termed Sequential-clustering Particle Annihilation via Discrepancy Estimation (SPADE). Specifically, SPADE follows a divide-and-conquer strategy: Adaptive clustering of particles via controlling their number-theoretic discrepancies and independent random matching in each cluster. Combining them together, we attempt to simulate the proton-electron couplings in 6-D and 12-D phase space. This talk is based on a series of works joint with Prof. Sihong Shao at Peking University.

Monte Carlo method and the isentropic Euler system

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This contribution presents our recent results on random isentropic Euler system. We apply Monte Carlo method combined with deterministic viscous finite volume (VFV) method. Our method is based on combination of several new concepts. We work with the dissipative weak solutions that can be seen as a universal closure of consistent approximations. Further, we apply the Strong law of large numbers and the Komlós theorem on strong converge of empirical averages of integrable functions. A structure-preserving and unconditionally convergent viscous finite volume (VFV) method is used to yield consistent approximations. We formulate theoretical results on the convergence and error estimates of the Monte Carlo estimates in the expectation. The error estimates for the expectation of Cesáro average verify the strong convergence of the Monte Carlo finite volume method. Theoretical results are illustrated by our extensive simulations of well-known instability problems, namely the Kelvin-Helmholtz problem and Richtmyer-Meshkov problem.

This is a joint work with E.Feireisl (Prague), H.Mizerová (Bratislava) and Mária Lukáčová-Medvid'ová (Mainz). It has been supported by the German Research Foundation (DFG) under the grant SPP 2410, Project number 525853336.

A new understanding of grazing limit

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The grazing limit of the Boltzmann equation to the Landau equation is well-known and has been justified by using cutoff near the grazing angle together with some suitable scaling. In this talk, we will provide a new understanding by only applying a simple scaling on the Boltzmann equation without introducing angular cutoff. We first establish a global well-posedness theory on the Boltzmann equation without angular cutoff in the parameter range $\gamma > -2s-3$ where γ is the exponent of the relative velocity and s represents the strength of angular singularity. The range is optimal in the following sense: it is a well-known fact that the dominant part of the Boltzmann operator behaves like a factional Laplace operator $-(-\Delta)^s$ which allows a singularity with exponent $-2s-3$ in the 3-dimensional space. Based on the global well-posedness, we justify the grazing limit $s \to 1$ for any $\gamma > -5$ that includes the Coulomb potential $\gamma = -3$. As a byproduct, the Landau equation is globally well-posed for any $\gamma > -5$. This is a joint work with Prof. Tong Tang.

Parallel Session D4 (CT)

Contributed Talks
East Middle Hall-Building 2-303,
11:10-18:05

On the convergence of an operator-splitting Lax-Friedrichs scheme for the Burgers-Hilbert equation

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In this work, we analyze the convergence of an explicit Lax-Friedrichs scheme to a weak solution of one-dimensional scalar balance laws with a singular integral operator as a source term. Our analysis mainly considers the Cauchy problem for the Burgers-Hilbert equation. The Burgers-Hilbert equation acts as a model problem for some two-dimensional incompressible fluid problems without viscosity such as free boundary dynamics of vortex patches and perturbations of shear flow. It is well known that any monotone finite volume scheme for scalar conservation laws is contractive in $L^1(\mathbb{R})$, while the source term Hilbert transform operator is a linear isometry from $L^2(\mathbb{R})$ onto itself and a bounded linear operator on L^2 but not on L^1. Thus the two sides of the Burgers-Hilbert equation are characterized by quite distinct behaviours and therefore the incompatibilities between the topologies used in the analysis of conservation laws and the source term make it very difficult to develop accurate and fast numerical methods. This dichotomy between the two sides of the equation brought us to treat our scheme as an operator-splitting approach here. Finally, by using a Lax-Wendroff type result under a suitable CFL condition, we prove that our numerical scheme converges to a weak solution of the Burgers-Hilbert equation. The results of the analysis are illustrated by a series of numerical simulations in the end to validate the results.

Hyperbolic viscous flow using quaternion fields

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This talk is concerned with representation issues associated with the numerical solution of a unified mathematical model of continuum mechanics, due to Godunov, Peshkov, and Romenski, which **can describe ideal fluids, viscous fluids and elastoplastic solids as special cases of a general continuum** [1,2]. The different regimes are characterized solely by the choice of material parameters and the resulting PDE system is of hyperbolic nature, with clearly defined finite wave speeds, in contrast to the standard formulation of viscous fluids via the Navier–Stokes equations.

The description of such a general continuum hinges on the evolution of a matrix-valued field called distortion, which is a generalization of the inverse deformation gradient in solid mechanics. In the fluid regime, this quantity can no longer be recovered as a gradient of displacements and encodes very rich information, in particular due to the different orientations that ideal fluid parcels can be found in. The fine features of the distortion field can be challenging (or outright impossible) to resolve with standard well-tested Finite Volume methods. Degenerate situations are routinely encountered where unphysical states are generated simply as a result of taking a convex combination of two data points.

We show how changing to an alternative representation of the same distortion field, obtained via polar decomposition, can be used to solve such discretization issues.

Instead of the original PDE system, one can instead evolve the rotational and stretch components of the distortion matrix separately, which allows the description of the rotational components through a **quaternion-valued partial differential equation**. We discuss the peculiarities of quaternion PDEs and some of the discretization strategies that they enable. We present numerical examples of high-Reynolds number simulations which could not be carried out with the previous formulation of the model.

[1] M. Dumbser, I. Peshkov, E. Romenski, O. Zanotti, High order ADER schemes for a unified first order hyperbolic formulation of continuum mechanics: Viscous heat-conducting fluids and elastic solids. *Journal of Computational Physics*, Vol. **314**, pp. 824–862, 2016.

[2] I. Peshkov, M. Dumbser, W. Boscheri, E. Romenski, S. Chiocchetti, M. Ioriatti, Simulation of non-Newtonian viscoplastic flows with a unified first order hyperbolic model and a structure-preserving semi-implicit scheme. *Computers and Fluids*, Vol. **224**, 30 June 2021, 104963.

On the chain rule property for the divergence operator with L^p vector field on $\mathbb{R}^2$

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Let $p \geq 1$ and let $\mathbf{v} \colon \mathbb{R}^d \to \mathbb{R}^d$ be a compactly supported vector field with $\mathbf{v} \in L^p(\mathbb{R}^d)$ and $\operatorname{div} \mathbf{v} = 0$ (in the sense of distributions). The *divergence operator* $\mathsf{A}(\rho) := \operatorname{div}(\rho\mathbf{v})$ is said to have the *chain rule property* if for any $\rho \in L^\infty(\mathbb{R}^d)$ the equality $\mathsf{A}(\rho) = 0$ implies that $\mathsf{A}(\beta \circ \rho) = 0$ for any $\beta \in C^1(\mathbb{R})$ such that $\beta(0) = 0$ and β' is bounded. The chain rule property is important in the context of the continuity equation, where it allows one to deduce uniqueness of weak solutions (via the renormalization property).

It is well-known that for $d \geq 3$ the chain rule property may fail even when $p = \infty$. However in the two-dimensional setting the chain rule property holds for $p = \infty$, but it may fail for $p < 2$. We show that for $d = 2$ the chain rule property holds whenever $p \geq 2$. We also discuss uniqueness of weak solutions of the Cauchy problem for the continuity equation.

The talk is based on a joint work with M.V. Korobkov.

This work was supported by the RSF project 24-21-00315.

Traveling waves to a parabolic-hyperbolic system with singular viscosity from chemotaxis

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This talk is concerned with the traveling wave solutions to a parabolic-hyperbolic system with singular viscosity. The system arises from a chemotaxis model with logarithmic sensitivity and fast diffusion, which possesses strong singularities for the sensitivity at zero-concentration of chemical signal, and for the diffusion at zero-population of cells, respectively. The main purpose is to show the existence of traveling waves connecting the singular zero-end-state, and particularly, to show the asymptotic stability of these traveling waves. The challenge of the problem is the interaction of two kinds of singularities involved in the model: one is the logarithmic singularity of the sensitivity; and the other is the power-law singularity of the diffusivity. To overcome the singularities for the wave stability, some new techniques of weighted energy method are introduced artfully.

Velocity averaging lemmas for general second-order equations

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This talk will be concerned with the regularity of the so-called velocity averages

$$\int_{\mathbb{R}_v} f(\mathbf{x}, v)\psi(v)\, dv.$$

Here, $\psi(v)$ is a given real function, and $f(\mathbf{x}, v)$ solves a general, multidimensional second-order equation of the form

$$\sum_{j=1}^{N} \mathbf{a}_j(v)\frac{\partial f}{\partial \mathbf{x}_j}(\mathbf{x}, v) - \sum_{j,k=1}^{N} \mathbf{b}_{jk}(v)\frac{\partial^2 f}{\partial \mathbf{x}_j \partial \mathbf{x}_k}(\mathbf{x}, v) = g(\mathbf{x}, v).$$

As P.-L. Lions, B. Perthame, and E. Tadmor [*J. Amer. Math. Soc.* **7** (1994) 169–191] ingeniously demonstrated, these results, commonly known as "velocity averaging lemmas," have profound consequences in the theory of degenerate parabolic-hyperbolic equations such as

$$\frac{\partial \varrho}{\partial t} + \sum_{j=1}^{N} \frac{\partial}{\partial x_j}\mathbf{A}_j(\varrho) - \sum_{j,k=1}^{N} \frac{\partial^2}{\partial x_j \partial x_k}\mathbf{B}_{jk}(\varrho) = 0.$$

We will present new compactness and Sobolev regularity principles for such velocity averages. Moreover, we also derive some criteria for technical hypotheses known as "non-degeneracy conditions", which generalize the examples of equations that satisfy such assumptions.

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Eulerian dynamics with nonlinear velocity alignment

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The Euler-alignment system describes the collective behaviors of animal swarms. In this talk, we introduce a new type of alignment interaction that depends nonlinearly on velocity. We explore the asymptotic flocking and alignment behaviors. Notably, the introduction of nonlinearity yields a spectrum of distinctive asymptotic behaviors. Moreover, we present a rigorous derivation of our system from a kinetic flocking model. This is joint work with McKenzie Black.

Exploring Innovative Strategies in Advection Equation Solutions by Leveraging Inverse Lax-Wendroff Procedure

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The advection equation in level set methods is traditionally solved using either explicit schemes (while accepting the inevitable stability conditions) or implicitly, preferably utilizing efficient compact schemes, such as the one discussed in [1]. The latter can be developed employing the Lax-Wendroff procedure, resulting in a numerical stencil that covers multiple spatial levels, but only two levels in time. While this approach is often accurate and efficient, it can lead to non-physical oscillations in the gradient of the solution, particularly in scenarios with discontinuous gradients.

The primary focus of this presentation is to introduce a novel and somewhat unconventional approach in which the inverse Lax-Wendroff procedure [2] is used instead. This results in numerical stencils that span multiple time levels and even include future time points. This methodology offers a solution to the oscillation issue present in its traditional counterpart for large Courant numbers.

[1] P. Frolkovič, N. Gajdošová Unconditionally stable higher order semi-implicit level set method for advection equations. *Applied Mathematics and Computation*, 466, 128460, 2024

[2] Shu, Chi-Wang. Inverse Lax-Wendroff Boundary Treatment: A Survey. *Communications in Mathematical Research* , 38 (3), 2022

Finite difference approximation of a fractional Burgers equation

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We study an initial value problem of

$$u_t + uu_x = \int_x^\infty \frac{u_y(t,y)}{(y-x)^{1/2}}dy, \text{ for } t > 0,\ x \in \mathbb{R}, \quad (22a)$$

$$u(0,x) = u_0(x) \text{ for } x \in \mathbb{R}, \quad (22b)$$

whose equivalent form of

$$\frac{\partial f}{\partial x} - \gamma f\frac{\partial f}{\partial \tau} = \beta\frac{\partial^{1/2} f}{\partial \tau^{1/2}},\ (\beta, \gamma > 0) \quad (23a)$$

$$f(t,0) = f_0(t) \quad (23b)$$

arises in a model of sound propagation in train tunnels taking into account of heredity, or wave propagation in solids affected by viscoelastic relaxation of materials. We assume that the support of u_0 is contained in $(-\infty, 0)$. This means that the support of f_0 is contained in $(0, \infty)$, which covers most of realistic situations. Under this assumption, $u(t, x)$ is determined by data in a bounded region for arbitrary point (t, x).

Then we apply finite difference approximation with upstream method to (22) treating spatial derivatives implicitly. Let $u_m := (u(m\Delta t, \Delta x), \cdots, u(m\Delta t, n\Delta x))^\top \in \mathbb{R}^n$ for each m, and write the discretized scheme in a symbolic way of

$$R_m u_{m+1} = u_m, \tag{24}$$

where R_m is an $n \times n$ matrix. We prove that the scheme retains fundamental properties as the original equation in a following way.

Theorem. *(i) R_m^{-1} is positive so that the scheme is monotone.*
(ii) $\rho(R_m^{-1}) < 1$, where $\rho(\cdot)$ denotes the spectral radius of a matrix.
(iii) $\|u_{m+1}\|_{L^\infty} \leq \|u_m\|_{L^\infty}$, i.e., the maximal principle holds.

In the presentation, we will compare numerical results obtained above with those computed by front tracking method. If time permits, front tracking method using piecewise linear functions will also be explained.

Stability and error estimates for conservation laws with discontinuous flux

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In this talk I will present and apply recent results pertaining stability for *conservation laws with discontinuous flux* and convergence rates of numerical methods approximating their solutions. These results will then be applied in the framework of uncertainty quantification for these types of equations.

In the first part of the talk I will give an overview of *conservation laws with discontinuous flux* which has been an active research area during the last several decades. Many selection criteria to single out a unique weak solution have been proposed in this context and several numerical schemes have been designed and analyzed in the literature. Surprisingly, the preexisting literature on *convergence rates* for such schemes is practically nonexistent. In this talk, focusing on so-called adapted entropy solutions, I will present the first-ever convergence rate results for finite volume and front tracking methods as well as a flux-stability result.

The second part of the talk is devoted to demonstrating applications of these stability and convergence rate estimates. First, I will present a general *uncertainty quantification framework* for conservation laws with discontinuous flux where the problem data (the initial datum, the flux, and the spatial dependency coefficient) are uncertain. A particular application constitutes two-phase reservoir simulations for reservoirs with spatially varying geological properties where the reservoir interfaces are only known up to certain statistical quantities.

[1] J. Badwaik and A. M. Ruf, *Convergence rates of monotone schemes for conservation laws with discontinuous flux*, SIAM J. Numer. Anal., 58, doi.org/10.1137/19M1283276, (2020), pp. 607–629.

[2] J. Badwaik, C. Klingenberg, N. H. Risebro, and A. M. Ruf, *Multilevel Monte Carlo finite volume methods for random conservation laws with discontinuous flux*, M2AN Math. Model. Numer. Anal., doi.org/10.1051/m2an/2021011, (2021).

[3] A. M. Ruf, *Flux-stability for conservation laws with discontinuous flux and convergence rates of the front tracking method*, IMA J. Numer. Anal., 42, doi.org/10.1093/imanum/draa101, (2021), pp. 1116–1142.

Active Flux Methods for Hyperbolic Systems Using the Method of Bicharacteristics

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We present a new Active Flux finite volume method based on the approximate evolution operator using bicharacteristics. It is a third order accurate finite volume method for hyperbolic conservation laws, which is based on the use of point values as well as cell average values of the conserved quantities. The resulting method is fully discrete and has a compact stencil in space and time. An important component of Active Flux methods is the evolution formula for the update of the point values. Here the method of bicharacteristics is used for the derivation of truly multidimensional approximate evolution operator that can be used for the evolution of point values in Active Flux methods. We introduce our recent results on the convergence analysis via dissipative measure-valued solutions and weak-strong uniqueness principle. Consequently, we show that the first order variant of the Active Flux finite volume method converges strongly to a strong solution of the multidimensional Euler system as long as the latter exists. Numerical results confirm accuracy and stability of the resulting Active Flux method for acoustics and the Euler equations of gas dynamics.

The present work was supported by the German Science Foundation (DFG) under SPP 2410: Hyperbolic Balance Laws: Complexity, Scales and Randomness, project number 525800857. It has been obtained in collaboration with C. Helzel (Düsseldorf), E. Chudzik (Düsseldorf) and Mária Lukáčová-Medvid'ová (Mainz).

Numerical solution of two dimensional nonlinear scalar conservation laws using compact implicit scheme

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We present a novel approach to numerically solve conservation laws in two-dimensional space through a compact implicit discretization method. We introduce a second-order accurate numerical scheme based on the finite volume method, with the compact implicit form of the discretization stencil which simplifies the solution of resulting algebraic systems. To find the solution, the fast sweeping method is embraced in the presentation.

We conduct numerical experiments for both linear and nonlinear conservation laws, represented by the advection equation and Burgers equation, showcasing the effectiveness of our approach in capturing related phenomena. To avoid unphysical oscillations in the case of nonsmooth solutions, we incorporate variants of Weighted Essentially Non-Oscillatory (WENO) approximations.

Our work extends previous research on compact implicit schemes, adapting them to finite volume methods suitable for hyperbolic equations.

Parallel Session E4 (CT)

Contributed Talks

East Middle Hall-Building 1-201,
11:10-18:05

High-Order Numerical Methods for Hyperbolic Systems of Nonlinear PDEs with Uncertainties

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In this talk, we present new high-order numerical methods for hyperbolic systems of nonlinear partial differential equations (PDEs) with uncertainties. The new approach is realized in the semi-discrete finite-volume framework and is based on fifth-order weighted essentially non-oscillatory (WENO) interpolations in (multi-dimensional) random space combined with second-order piecewise linear reconstruction in physical space. Compared with spectral approximations in the random space, the presented methods do not only achieve high accuracy in the random space, even in the generic case of discontinuous solutions, but are also essentially non-oscillatory as they do not suffer from the Gibbs phenomenon while still achieving high-order accuracy. The new methods are tested on several numerical examples for the Euler equations of gas dynamics and the Saint-Venant system of shallow-water equations. In the latter case, the methods are also proven to be well-balanced and positivity-preserving.

Multilevel strategy for hyperbolic conservation laws with uncertain initial data

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We discuss hyperbolic conservation laws with uncertain initial data. It is well known that deterministic hyperbolic conservation laws have discontinuities arising in finite time. For random hyperbolic conservation laws this can also lead to discontinuities in the stochastic direction. The interplay of the different scales makes it difficult to find an efficient method to approximate the stochastic moments.

In [1], a random hyperbolic conservation law is interpreted as a higher dimensional deterministic problem, which is approximated by a DG-scheme. In order to reduce the computational complexity, multiresolution based grid adaptation [2] is applied to the higher dimensional formulation. Since we are interested in the stochastic moments rather than the solution itself, solution-oriented grid adaptation with uniform thresholding is suboptimal in terms of efficiency. Therefore, in [1] we develop a multiresolution strategy with weighted thresholding and apply it to the higher dimensional formulation. In contrast to uniform thresholding, weighted thresholding takes into account the local information of a solution and its current probability density. This results in a goal-oriented grid adaptation with respect to the stochastic moments instead of the solution. Although the solution with weighted thresholding may look poor compared to a solution with uniform thresholding, we show that we preserve accuracy in the stochastic moments with significantly fewer cells.

However, the methods presented in [1] have to be solved in a higher dimensional setting which becomes infeasible for many random variables. In [3], we present a novel scheme by separating the spatial and stochastic scales and applying different grid adaptation methods to the two different scales. Similar to Monte Carlo methods, we obtain samples of the random hyperbolic conservation laws for fixed realizations of the random variable. Additionally, we determine solutions in the stochastic direction for fixed spatial variables. We use different grid adaptation strategies for the spatial and stochastic directions, respectively. The spatial grid is obtained by performing multiresolution based grid adaptation on the expected value of the solution, while we perform multiresolution analysis with weighted thresholding in the stochastic direction. The resulting adaptive grids in each direction determine the resolution in the other direction and is adapted to the local structure of the corresponding solution. Thus, we consider the interaction of the spatial and stochastic scales of a random hyperbolic conservation laws without solving a higher dimensional problem. In this talk we introduce the multilevel strategy and present numerical results and compare them with Monte Carlo methods and the deterministic formulation.

[1] M. Herty, A. Kolb and S. Müller. “Multiresolution analysis for stochastic hyperbolic conservation laws‘”, *IMA Journal of Numerical Analysis*, 2023

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[3] A. Kolb “Multiresolution-based grid adaptation for hyperbolic conservation laws with uncertain initial data“, *PhD dissertation, RWTH Aachen 2023*, DOI 10.18154/RWTH-2024-00677

Recent results on asymptotic behavior of almost periodic solutions of stochastic parabolic-hyperbolic conservation laws

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We present some recent results on the asymptotic behavior of almost periodic solutions to stochastic conservation laws and, more generally, degenerate parabolic-hyperbolic equations. Two types if stochastic perturbations are considered: forcing and rough-flux. The part concerning the forcing stochastic source is from joint works with Claudia Espitia and Daniel Marroquin. The part concerning stochastic rough-flux is from a joint project with Rui Jin Yachun Li and João Nariyoshi.

SPDEs as singular stochastic dynamical system in Hilbert space and its stability

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In this talk, we discuss some results of the transport-type SPDEs. These equations can be viewed as the singular SDEs in Sobolev space. We establish a local well-posedness result of its pathwise solution. And then, we consider the impact of noise on both preventing blow-up and its stability. On one hand, we identify a family of noises such that blow-up can be prevented with probability 1, guaranteeing the existence and uniqueness of global solutions almost surely. On the other hand, we introduce a notion of stability of exiting time and we show that one cannot improve the stability of the exiting time and simultaneously improve the continuity of the dependence on initial data.

Non-locality in Stochastic Fluid PDEs: A Study on Pseudo-differential Noise and Mean-Field

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In this talk, I will discuss SPDEs of fluid type featuring pseudo-differential noise (non-local in the spatial variable) and mean-field (non-local in the sample variable), respectively. In the case of pseudo-differential noise, we have uncovered certain cancellation properties inherent to pseudo-differential operators, which play pivotal roles in establishing the existence of solutions. Concerning mean-field dynamics, unlike classical SDEs/SPDEs, relying solely on stopping time techniques does not guarantee uniqueness. We have identified a new localized topology that enables us to address this issue. Additionally, we have established abstract frameworks for each of these scenarios.

Non-uniqueness in law of Leray solutions to 3D forced stochastic Navier-Stokes equations

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This talk concerns the forced stochastic Navier-Stokes equation driven by additive noise in the three dimensional Euclidean space. By constructing an appropriate forcing term, we show that there exist distinct Leray solutions in the probabilistically weak sense. In particular, the joint uniqueness in law fails in the Leray class. The non-uniqueness also displays in the probabilistically strong sense in the local time regime, up to stopping times. Furthermore, we discuss the optimality from two different perspectives: sharpness of the hyper-viscous exponent and size of the external force. As a consequence, one derives that the Lions exponent is the sharp viscosity threshold for the uniqueness/non-uniqueness in law of Leray solutions.

Existence and Asymptotic Limits for Strain-Gradient Viscoelasticity

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In this work, we study the following system of viscoelasticity with second-order gradients, or strain-gradient viscoelasticity

$$\begin{cases} \partial_t u^{\nu,\delta} = \mathrm{div} S(F^{\nu,\delta}) + \nu\Delta u^{\nu,\delta} - \delta\nabla\Delta F^{\nu,\delta} \\ \partial_t F^{\nu,\delta} = \nabla u^{\nu,\delta} \\ \mathrm{curl} F^{\nu,\delta} = 0 \end{cases} \tag{25}$$

on the periodic domain $\mathbb{T}^d$ for $d = 2, 3$, and with initial data $(u_0^{\nu,\delta}, F_0^{\nu,\delta}) \in L^2(\mathbb{T}^d) \times L^p(\mathbb{T}^d) \cap H^1(\mathbb{T}^d)$. The unknowns of the system are the velocity $u^{\nu,\delta}$ and deformation gradient $F^{\nu,\delta}$. The coefficients ν and δ are the viscosity and dispersion coefficients, respectively, which are assumed to be positive constants.
This system is a diffusive-dispersive regularization of the equations of hyperelasticity where the elastic energy $S(F) = \frac{\partial W}{\partial F}(F)$ is assumed to be semiconvex. We first establish global existence for initial data in the regularity class $(u_0^{\nu,\delta}, F_0^{\nu,\delta}) \in L^2(\mathbb{T}^d) \times H^1(\mathbb{T}^d)$. Using compactness methods, we then show convergence from strain-gradient viscoelasticity to viscoelasticity of Kelvin-Voigt type

$$\begin{cases} \partial_t u^{\nu} = \mathrm{div} S(F^{\nu}) + \nu\Delta u^{\nu} \\ \partial_t F^{\nu} = \nabla u^{\nu} \\ \mathrm{curl} F^{\nu} = 0 \end{cases} \tag{26}$$

as the dispersion coefficient goes to zero, $\delta \to 0$. In the two-dimensional case, we prove, under some additional assumptions, the following rate of convergence of order $\mathcal{O}(\delta^{\frac{1}{2}})$

$$\sup_{t\in(0,T^*)} \left(\int |F^{\nu,\delta} - F^{\nu}|^r dx \right)^{\frac{1}{r}} \le \tilde{C}\delta^{\frac{1}{2}} \exp\left\{ 2 - 2\exp(-C_{T^*}) \right\}^{\frac{1}{r}}, \quad 1 < r < 2.$$

which is derived following the approach of Yudovich as adopted by Chemin in the proof of the inviscid limit for the two-dimensional Navier-Stokes equations with bounded voritcity. Finally, we also investigate the limit as the viscosity coefficient, ν, goes to zero.
This is a joint work with Stefano Spirito (University of L'Aquila) and Athanasios Tzavaras (KAUST).

Very weak solutions to the 2D Monge–Ampère equation

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In this talk, we first review the previous construction of very weak solutions to the two-dimensional Monge–Ampère equation by the approach of convex integration. Then we talk about our recent result: for any $\beta < \frac{1}{3}$, we prove that very weak solutions with regularity $C^{1,\beta}$ are dense in the space of continuous functions. This talk is based on the joint work with Profs. Dominik Inauen and Jonas Hirsch from Leipzig University.

Recent progress on the Schrödinger map equation

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In this talk, we will discuss some results of the Schrödinger map equation, a geometric partial differential equation by considering its evolution for polygonal curves in different geometrical settings. The equation is a special case of the famous Landau–Lifshtiz equation for ferromagnetism and its equivalent form in the Euclidean space describes the evolution of a vortex filament in a real fluid, known as the vortex filament equation.

The dynamics of these equations for polygonal initial data when solved numerically exhibit several interesting characteristics of real fluids, e.g., the axis switching phenomenon and multifractality, often associated with turbulence. On the other hand, the algebraic construction of these solutions not only supports the numerical evolution but also indicates randomness. I will present some recent results, in particular, the case of helical-shaped vortices and curves in the hyperbolic space, and show that this unusual behaviour (randomness) resulting from a differential equation is indeed appears as a generic phenomenon.

A part of the talk is a work in collaboration with Luis Vega (BCAM, UPV/EHU) and Francisco de la Hoz (UPV/EHU).

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Green's function for solving initial-boundary value problem of evolutionary partial differential equations

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We propose a new method to solve the initial-boundary value problem for hyperbolic-dissipative partial differential equations based on the spirit of LY algorithm [1]. The new method can handle more general domains than that of LYs'. We convert the evolutionary PDEs into the elliptic PDEs by the Laplace transformation. Using the Laplace transformation of the fundamental solutions of the evolutionary PDEs and the image method, we can construct Green's functions for the corresponding elliptic PDEs. Finally, we obtain Green's functions for the evolutionary PDEs by inverting the Laplace transformation. As a consequence, we establish Green's functions for some basic PDEs such as the heat equation, the wave equation and the damped wave equation, in a half space and a quarter plane with various boundary conditions. On the other hand, the structure of hyperbolic-dissipative PDEs means its fundamental solution is non-symmetric and hence the image method does not work. We utilize the idea of Laplace wave train introduced by Liu and Yu in [2] to generalize the image method. Combining this with the notions of Rayleigh surface wave operators introduced in [3], we are able to obtain the complete representations of Green's functions for the convection-diffusion equation and the drifted wave equation in a half space with various boundary conditions.

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Emergence of biological transportation networks as a self-regulated process

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We propose a mesoscopic modeling framework for optimal transportation networks with biological applications. The network is described in terms of a joint probability measure on the phase space of tensor-valued conductivity and position in physical space. The energy expenditure of the network is given by a functional consisting of a pumping (kinetic) and metabolic power-law term, constrained by a Poisson equation accounting for local mass conservation. We establish convexity and lower semicontinuity of the functional on appropriate sets. We then derive its gradient flow with respect to the 2-Wasserstein topology on the space of probability measures, which leads to a transport equation, coupled to the Poisson equation. To lessen the mathematical complexity of the problem, we derive a reduced Wasserstein gradient flow, taken with respect to the tensor-valued conductivity variable only. We then construct equilibrium measures of the resulting PDE system. Finally, we derive the gradient flow of the constrained energy functional with respect to the Fisher-Rao (or Hellinger-Kakutani) metric, which gives a reaction-type PDE. We calculate its equilibrium states, represented by measures concentrated on a hypersurface in the phase space.

Parallel Session F4 (SF)

PhD Student Forum

East Middle Hall-Building 1-202, 14:00-18:05

An entropy stable and material contact surface preserving scheme for compressible multi-component flows

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Numerical schemes based on entropy conservation/stability have been the focus of research in the recent past and are perceived to be essential for convergence to the physically relevant solutions (see, for example, Tadmor [1] and Ismail & Roe [2]). A recent extension of such schemes to the multi-component Euler equations using a fully conservative model is presented by Gouasmi et al [3]. Recently we introduced a novel entropy conservative Euler scheme [5]. In this work, utilize this stragegy as the foundation for developing an entropy conservative/stable scheme for multi-component flows.

Most Godunov-type schemes suffer from spurious pressure oscillations at material contact surfaces, while simulating multi-component flows. One of the strategies introduced to overcome this artifact is the gamma-model proposed by Abgrall [4], which utilizes a non-conservative gamma-based formulation. In the current work, we extend the methodology of obtaining entropy conservative fluxes of Bahuguna and Raghurama Rao [5] to the multi-component flows, with the gamma-based model. An appropriate diffusion is added to a contact discontinuity preserving central flux (adding diffusion based on Rankine-Hugonoit conditions to preserve steady discontinuities exactly [6]) such that entropy conservation equation is satisfied for the semi-discrete case. Based on the partial fractions, a material contact surface detecting sensor is used to switch back to the contact discontinuity preserving flux. Thus the spurious pressure oscillations across material contact surfaces are avoided while maintaining entropy stability at all other regions. The scheme is tested for accuracy and robustness on various bench-mark test cases.

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Discrete-velocity-direction models of BGK-type with minimum entropy

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In this work, we develop a discrete-velocity-direction model (DVDM) with collisions of BGK-type for simulating gas flows, where the molecular motion is confined to some prescribed directions but the speed is still a continuous variable in each orientation. The discrete equilibriums in the collision terms are determined by the minimum entropy principle subject to the constraints of conservation laws. We also show that the discrete equilibriums can be efficiently obtained by solving a convex optimization problem. With this DVDM, we develop spatial-time submodels by incorporating 1-D treatments in each direction. These spatial-time submodels are novel multidimensional versions corresponding to the 1-D approaches. Numerical tests with a series of 1-D and 2-D flow problems show the efficiency of the DVDM.

Mean field error estimate of the random batch method for large interacting particle system

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The random batch method (RBM) proposed in [Jin et al., J. Comput. Phys., 400(2020), 108877] for large interacting particle systems is an efficient with linear complexity in particle numbers and highly scalable algorithm for N-particle interacting systems and their mean-field limits when N is large. We consider in this work the quantitative error estimate of RBM toward its mean-field limit, the Fokker-Planck equation. Under mild assumptions, we obtain a uniform-in-time $O(\tau^2 + 1/N)$ bound on the scaled relative entropy between the joint law of the random batch particles and the tensorized law at the mean-field limit, where τ is the time step size and N is the number of particles. Therefore, we improve the existing rate in discretization step size from $O(\sqrt{\tau})$ to $O(\tau)$ in terms of the Wasserstein distance.

Quantum simulation of Maxwell's equations via Schrödingerisation

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We present quantum algorithms for electromagnetic fields governed by Maxwell's equations. The algorithms are based on the Schrödingerisation approach, which transforms any linear PDEs and ODEs with non-unitary dynamics into a system evolving under unitary dynamics, via a warped phase transformation that maps the equation into one higher dimension. In this paper, our quantum algorithms are based on either a direct approximation of Maxwell's equations combined with Yee's algorithm, or a matrix representation in terms of Riemann-Silberstein vectors combined with a spectral approach and an upwind scheme. We implement these algorithms with physical boundary conditions, including perfect conductor and impedance boundaries. We also solve Maxwell's equations for a linear inhomogeneous medium, specifically the interface problem. Several numerical experiments are performed to demonstrate the validity of this approach. In addition, instead of qubits, the quantum algorithms can also be formulated in the continuous variable quantum framework, which allows the quantum simulation of Maxwell's equations in analog quantum simulation.

Keywords: Maxwell's equations, quantum algorithm, Schrödingerisation method, boundary and interface conditions, continuous-variable quantum system

Convergence of a second-order scheme for non-local conservation laws

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In this talk, we discuss the convergence analysis of a second-order numerical scheme for traffic flow models that incorporate non-local conservation laws to capture the interaction between drivers and the surrounding density of vehicles. We combine a MUSCL-type spatial reconstruction with strong stability preserving Runge-Kutta time-stepping to devise a fully discrete second-order scheme. The resulting scheme is shown to converge to a weak solution by establishing the maximum principle, bounded variation estimates and L^1-Lipschitz continuity in time. Further, using a space-step dependent slope limiter, we prove its convergence to the entropy solution. We also propose a MUSCL-Hancock type second-order scheme which requires only one intermediate stage unlike the Runge-Kutta schemes and is easier to implement. The performance of the proposed second-order schemes in comparison to a first-order scheme is demonstrated through several numerical experiments.

This is a joint work with Dr. Sudarshan Kumar K. (IISER Thiruvananthapuram, Thiruvananthapuram, India) and Prof. G. D. Veerappa Gowda (TIFR-CAM, Bangalore, India).

A new central compact finite difference scheme with high spectral resolution for dispersive equations

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This work introduces a new class of central compact schemes, drawing inspiration from established cell-node and cell-centered compact finite difference schemes [1,2,3,4], which offer enhanced spectral resolution for solving the dispersive wave equation. Our approach utilizes both function values at cell nodes and cell centers to compute third-order spatial derivatives at the nodes. To calculate spatial derivatives at the cell centers, we employ a technique involving half-shifting indices within the formula initially devised for node calculations. Unlike conventional compact interpolation schemes, our method effectively circumvents the introduction of transfer errors. We determine finite difference coefficients using either a Taylor series expansion-based approach or optimized least squares-based methods. Third-order TVD Runge–Kutta scheme [5] is used to evaluate time derivative. Through systematic Fourier analysis and numerical experiments, we observe exceptional characteristics such as high order, superior resolution, and low dissipation in our methods. Computational results further demonstrate the efficacy of high-order compact schemes, particularly in tackling problems involving third-derivative terms.

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Propagation of chaos in path spaces via information theory

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In this talk, we will discuss a new approach to study the mean-field limit of the stochastic interacting particle systems via tools from information theory. The main technique is the application of the data processing inequality, which enables one to only estimate the difference between drifts of the particle system and the mean-field Mckean stochastic differential equation. This point is particularly useful for second-order systems, overcoming the usual degeneracy of noises and avoiding using the usual hypocoercivity technique. The convergence rate for second-order systems is independent of the particle mass. This is joint work with Yuliang Wang and Lei Li.

Stability Analysis of Hyperbolic Quadrature Method of Moments for Kinetic Equations

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This work gives a stability analysis of a quadrature method of moments (called HyQMOM) for the one-dimensional BGK equation. The method is well defined for all realizable moments and infers unclosed terms by constructing higher order orthogonal polynomials. It is positivity preserving and shows good simulation performance in numerical tests. However, a theoretical analysis is largely absent but desirable. We rigorously prove strict hyperbolicity of the HyQMOM-derived moment systems by revealing the relationship between moment closure and characteristic polynomial of the resultant first-order PDE. We further show that the system preserves dissipative property of the kinetic equation if the HyQMOM closure is affine invariant. The orthogonal polynomials endowed with the HyQMOM enables development of provable realizability-preserving schemes with the HyQMOM reconstruction of finitely-supported distributions.

Parallel Session G4 (SF)

PhD Student Forum
East Middle Hall-Building 1-301,
14:00-18:05

Existence and Stability of Stationary Solutions of Inflow Problem for bipolar quantum Navier-Stokes-Poisson equations

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We are interested in the asymptotic behavior of solutions to an inflow problem for the one-dimensional bipolar quantum Navier-Stokes-Poisson equations in the first quadrant. For the initial boundary value problem, first we show the existence by the application of the manifold theory and the center manifold theorem provided the boundary strength is small enough. After that, we rely on energy estimates to provide the stability result of the stationary solution based upon some assumptions.

Stability of stationary compressible Navier-Stokes flows on the 3D whole space

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We consider the initial value problem for the compressible Navier-Stokes equation in the 3D whole space:

$$\begin{cases} \partial_t \rho + \operatorname{div}(\rho v) = 0, \\ \partial_t(\rho v) + \operatorname{div}(\rho v \otimes v) = \mu \Delta v \\ \quad + (\mu + \mu')\nabla \operatorname{div} v - \nabla \left(P(\rho)\right) + \rho F(x), \\ (\rho, v)|_{t=0} = (\rho_0, v_0), \quad \lim_{|x|\to\infty} (\rho, v) = (\rho_\infty, 0). \end{cases} \tag{27}$$

Here $t \geq 0$, $x \in \mathbb{R}^3$, $v = (v_1, v_2, v_3)$ is the fluid velocity, ρ is the fluid density, ρ_∞ is a given positive constant, P is a given pressure, μ and μ' are given viscosity coefficients and $F = (F_1, F_2, F_3)$ is a given stationary external force. We assume that μ and μ' are constants that satisfy $\mu > 0$ and $2\mu/3 + \mu' \geq 0$, and P is a smooth function of ρ in a neighborhood of ρ_∞ with $P'(\rho_\infty) > 0$.

Shibata and Tanaka [1] proved the existence of the stationary solution (ρ^*, v^*) which satisfy the decay estimate

$$|v^*(x)| \lesssim \frac{1}{|x|}, \quad |\nabla v^*(x)| \lesssim \frac{1}{|x|^2}, \quad |\rho^*(x) - \rho_\infty| \lesssim \frac{1}{|x|^2} \quad \text{as} \quad |x| \to \infty. \tag{28}$$

They also proved the existence of the global solution (ρ, v) when the initial perturbation $(\rho_0 - \rho^*, v_0 - v^*)$ belongs to $H^3(\mathbb{R}^3)$ and is small in $H^3(\mathbb{R}^3)$ norm. In [2], Shibata and Tanaka then derived the decay rates of the perturbations

$$\begin{aligned} &\|(\rho - \rho^*, v - v^*)(t)\|_{\dot{H}^1} \\ &\lesssim_\epsilon (1+t)^{-\frac{1}{2}+\epsilon} \|(\rho_0 - \rho^*, v_0 - v^*)\|_{L^{\frac{6}{5}} \cap H^3}, \end{aligned} \tag{29}$$

where $\epsilon > 0$ is an arbitrary constant.

The aim of this talk is to study the decay rate of the perturbation around the stationary solution. In this talk, we show the following result related to the above problem.

Theorem. *Let (ρ^*, v^*), (ρ, v) be as above, with*

$$\|(\rho_0 - \rho^*, v_0 - v^*)\|_{\dot{B}^{\frac{1}{2}}_{2,\infty} \cap \dot{H}^3} + \|F\|_{\dot{B}^{-\frac{3}{2}}_{2,\infty} \cap \dot{H}^3}$$

sufficiently small. If the initial perturbation $(\rho_0 - \rho^*, v_0 - v^*) \in L^p$ for some $1 \leq p \leq 2$, then the decay estimate

$$\begin{aligned} &\|(\rho - \rho^*, v - v^*)(t)\|_{\dot{H}^s} \\ &\lesssim_s (1+t)^{-\frac{s}{2} - \frac{3}{2}\left(\frac{1}{p} - \frac{1}{2}\right)} \|(\rho_0 - \rho^*, v_0 - v^*)\|_{L^p \cap H^3} \end{aligned}$$

holds for $-3/2 < s < 3/2$ with $s/2 + 3/2(1/p - 1/2) > 0$ or $s = 0$ and $p = 2$. In addition, if $p = 1$, then the time decay estimate (7) is optimal.

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Variational structure and two-dimensional subsonic jet flows for compressible Euler system with general incoming flows

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In this talk, we show the well-posedness of compressible subsonic jet flows for two-dimensional steady Euler system with general incoming horizontal velocity as long as the flux is larger than a critical value. One of the key observations is that the stream function formulation for two-dimensional compressible steady Euler system enjoys a variational structure even when the flows have nontrivial vorticity, so that the framework developed by Alt, Caffarelli and Friedman can be adapted to study the jet problem, which is a Bernoulli type free boundary problem. A major technical point to analyze the jet flows is that the inhomogeneous terms in the rescaled equation near the free boundary are always small, even when the vorticity of the flows is big. This is a joint work with Wenhui Shi, Lan Tang and Chunjing Xie.

Asymptotic preserving decomposed method for the radiative transfer system

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The radiation magnetohydrodynamics (RMHD) plays an important role in the high temperature flow systems, inertial confinement fusion and astrophysics. The main challenge is that the radiation travels at the speed of light while the magnetohydrodynamics changes with the time scale of the fluid. The time scales of these two processes can vary dramatically. In order to use mesh sizes and time steps that are independent of the speed of light, asymptotic preserving (AP) schemes in both space and time are desired. We develop AP schemes for the RMHD system and the frequency-dependent radiative transfer system. The performance of the semi-implicit method is presented, for both optically thin and thick regions.

a-contraction theory for general viscous systems of conservation laws

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The a-contraction theory is a recent energy based method developed to tackle the stability and asymptotic limits for general solutions to conservation laws.

In this talk, I will focus on the time asymptotic stability of shocks for viscous conservation laws.

I will describe how the a-contraction theory, first developed in this context to the compressible Navier-Stokes equation, can be extended to such general situations. This is a joint work with Young-Sam Kwon and Alexis Vasseur.

Interactions between elementary waves and weak discontinuity in two-layer blood flow through artery

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Blood flow through arterial circulation can be characterized by fluid transport in flexible tubes and modeled mathematically using the conservation of mass and momentum. A one-dimensional model for two-layer blood flow with different blood velocities and the same constant density in each layer derived from the Euler equations of gas dynamics by taking the vertical average across each layer. This work presents interactions of elementary waves with a weak discontinuity for the quasilinear 3×3 system of conservation laws governing the two-layer blood flow in arteries. Exploiting elementary waves as a single-parameter curve, we study the Riemann solution uniquely and consequently establish the condition on initial data for the existence of a solution to the Riemann problem. Furthermore, we discuss the evolution of weak discontinuity waves and subsequently derive their amplitudes; in what follows, we investigate the interactions of weak discontinuity with contact discontinuity and shocks. Finally, a series of numerical tests have been performed to understand the impact of shock strength and the initial data on the amplitudes of reflected and transmitted waves and the jumps in shock acceleration.

Conservation laws and invariant solutions for one-dimensional shallow water magnetohydrodynamics equations using Lie group theory

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This article focuses on constructing exact solutions for a system of hyperbolic partial differential equations that govern one-dimensional shallow water magnetohydrodynamics equations. By performing the local symmetry analysis of the governing system, we establish the one-dimensional optimal system of subalgebras and generate several exact solutions through symmetry reductions. Additionally, employing direct multipliers, we illustrate various conserved quantities of the system, leading to associated nonlocally related potential systems. Furthermore, we introduce potential systems and inverse potential systems (IPS) to categorize nonlocal symmetries of the governing system. We develop non-trivial exact solutions for the model through the nonlocal symmetries of IPS. Finally, we apply one of the obtained solutions to analyze the evolutionary properties of weak discontinuity and wave interaction.

Transition threshold for the 2-D Couette flow in whole space via Green's function

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In this talk, we investigate the transition threshold problem concerning the 2-D Navier-Stokes equations in the context of Couette flow $(y, 0)$ at high Reynolds number Re in whole space. By utilizing Green's function estimates for the linearized equations around Couette flow, we initially establish refined dissipation estimates for the linearized Navier-Stokes equations with a precise decay rate $(1+t)^{-1}$. As an application, we prove that if the initial perturbation of vorticity satisfies

$$\|\omega_0\|_{H^1 \cap L^1} \leq c_0 \nu^{\frac{3}{4}}$$

for some small constant c_0 independent of the viscosity ν, then we can reach the conclusion that the solution remains within $O\left(\nu^{\frac{3}{4}}\right)$ of the Couette flow. This is joint work with Weike Wang.

The linear ill-posedness of the Prandtl equation of Non-Newtonian flow

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The well-posedness of the Prandtl equation has been an open question in many different settings, as well as the form of the Non-Newtonian flow. In this PhD student forum, we would like to show some results about the linear ill-posedness of the Prandtl euqation of Non-Newtonian flow in Sobolev type spaces. The basic idea is as follows: First we consider the destabilization of linearized system where shear flow is independent of time. Then we analyse the original equation considering the variations of shear flow. Our result shows the linear ill-posedness in certain space, whose main novelty is to show the ill-posedness condition related to power index n of Non-Newtonian flow. We finally show some special case with certain power index n.